Conformal Defects and Emergent Supersymmetry

based on ArXiV 2012.00018

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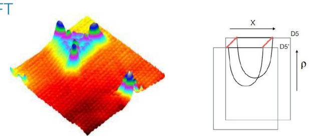
What are conformal defects?

We can add extended objects to a *d*-dimensional CFT



They break the (super)conformal group. When they preserve a conformal subgroup, they are called conformal defects:

$$SO(d+1,1) \to SO(p+1,1) \times SO(d-p)$$

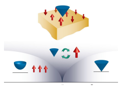
The defect has dimension *p* and also preserves rotations around the defect *SO(d - p)*.

There is *no conserved stress-tensor* on the defect.

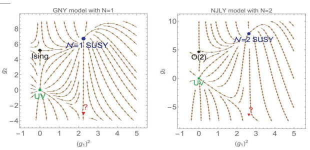
Emergent Supersymmetry

Problem: SUSY is beautiful, but has not been experimentally confirmed yet.

However, in condensed matter systems there are hints for emergent SUSY, for example on the boundary of topological superconductors



or it emerges in conformal fixed points



Boundaries in 3d N=2 SCFTs

In recent work we considered a 3d theory with N=2 SUSY with a boundary. There are two possibilities

3d N = 2 (OSP(2|4))

2d N = (2,0)	2d N = (1,1)
R-symmetry	No R-symmetry
Can't be extended to 4d	Can be extended to 4d
$\Phi_{3d} \to \hat{\Phi} + \theta\hat{\Psi} + \cdots$	$\Phi_{3d} \to \hat{\Phi} + \cdots$

We calculated several correlators

$\langle \phi_1 \hat{\Phi}_2 \rangle, \quad \langle \phi_1 \hat{\Psi}_2 \rangle$	fixed by SCF symmetry
$\langle \phi_1 \hat{O}_2 \hat{O}_3 \rangle$	no crossing
$\langle \phi_1 \bar{\phi}_2 \rangle, \quad \langle \phi_1 \phi_2 \rangle$	can be bootstrapped!

and studied the influence of a free bulk on the boundary operator spectrum.

For the N = (1,1) boundary, we went across dimensions.

The ε-expansion

We can bootstrap the 2-pt functions for d = 4 - ε. The crossing equations are

$$F^{\phi\bar{\phi}}_{\mathrm{Id}}(\xi) + \sum_n c_n F^{\phi\bar{\phi}}_{\Delta_n}(\xi) = \sum_n \mu_n \hat{F}^{\phi\bar{\phi}}_{\hat{\Delta}_n}(\xi)$$

$$\sum_n d_n F^{\phi\phi}_{\Delta_n}(\xi) = \sum_n \rho_n \hat{F}^{\phi\phi}_{\hat{\Delta}_n}(\xi)$$

and each dimension and coefficient is expanded in ε, for example

$$\Delta_\phi = \frac{d-2}{2} + \Delta^{(1)}_\phi \epsilon + \Delta^{(2)}_\phi \epsilon^2 + \ldots \quad c_n = c^{(0)}_n + c^{(1)}_n \epsilon + c^{(2)}_n \epsilon^2 + \ldots$$

At 0th order in ε, only one block on each side contributes:

$$F^{\phi\bar{\phi}}_{\mathrm{Id}}(\xi) = \hat{F}^{\phi\bar{\phi}}_{(d-2)/2}(\xi) \qquad F^{\phi\phi}_{d-2}(\xi) = \hat{F}^{\phi\phi}_{(d-2)/2}(\xi)$$

At 1st order, we get infinite blocks. We can solve the crossing equation using discontinuities.
We relate $\rho_n = \pm\mu_n$ and get

$$c^{(1)}_0 = \Delta^{(1)}_\phi - \hat{\Delta}^{(1)}_0, \qquad c^{(1)}_{n\geq 1} = 0, \qquad d^{(1)}_0 = 0, \qquad \mu^{(1)}_0 = \rho^{(1)}_0 = 0\,,$$

$$\mu^{(1)}_n = s(-1)^n \rho^{(1)}_n = s(-1)^n d^{(1)}_n = \frac{(n-1)!}{2^{n-1}(2n-1)!!}\Delta^{(1)}_\phi, \qquad n \geq 1\,.$$

At 2nd order, we need different techniques...

We compared with the 3d N=2 WZ model with a boundary and found perfect agreement.

The Conformal Bootstrap

In a CFT, 2- and 3-point (scalar) correlators are conformally fixed:

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \frac{\delta_{\Delta_1,\Delta_2}}{x_{12}^{2\Delta_1}} \quad \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle = \frac{f_{123}}{x_{12}^{\Delta_1+\Delta_{23}} x_{23}^{\Delta_2+\Delta_{31}} x_{31}^{\Delta_3+\Delta_{12}}}$$

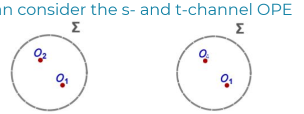
4-point correlators depend on cross-ratios:

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle = \frac{g(u,v)}{x_{12}^{\frac{1}{2}(\Delta_1+\Delta_2)} x_{34}^{\frac{1}{2}(\Delta_3+\Delta_4)}} \left(\frac{x_{24}}{x_{14}}\right)^{\frac{1}{2}\Delta_{12}} \left(\frac{x_{14}}{x_{13}}\right)^{\frac{1}{2}\Delta_{34}}$$

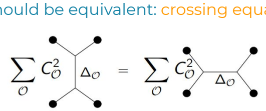
$g(u,v)$ can be expanded in an infinite sum of conformal blocks using the bulk OPE

$$O_1(x_1) O_2(x_2) \sim c_{123}\, |x^\mu_{12}|^{\Delta_3-\Delta_1-\Delta_2}\, O_3(x_2) + \ldots$$

We can consider the s- and t-channel OPE



which should be equivalent: crossing equation



Now we solve for the CFT data using numerics or analytical methods.

The Defect Bootstrap

Bulk operators close to the defect can acquire a one-point function which is conformally fixed

$$\langle \mathcal{O} \rangle = \frac{a_{\mathcal{O}}}{|2x^\perp|^\Delta}$$

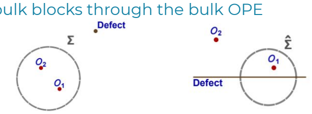
2-point bulk correlators depend on cross-ratios:

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \frac{G(\xi,\phi)}{|2x_1^\perp|^{\Delta_1} |2x_2^\perp|^{\Delta_2}}$$

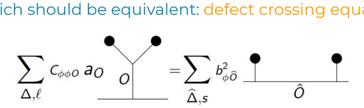
$G(\xi,\phi)$ can be expanded in an infinite sum of defect blocks using the defect OPE

$$O(x^a, x^i) \sim b_{O\hat{O}}\, |x^i|^{\widehat{\Delta}_2-\Delta_1}\, \widehat{O}(x^a) + \ldots$$

or in bulk blocks through the bulk OPE



which should be equivalent: defect crossing equation



The defect theory is non-unitary, so we can only solve the crossing equation analytically.

Going across dimensions

We extended the N = (1,1) boundary to a 4d boundary and calculated the 2-pt blocks

Boundary blocks

$$\hat{F}^{\phi_1\bar{\phi}_2}_{\hat{\Delta}}(\xi) = \hat{f}_{\hat{\Delta}}(\xi) + \frac{\hat{\Delta}}{2(2\hat{\Delta}-d+3)}\hat{f}_{\hat{\Delta}+1}(\xi)$$

$$\hat{F}^{\phi_1\phi_2}_{\hat{\Delta}}(\xi) = \hat{f}_{\hat{\Delta}}(\xi) - \frac{\hat{\Delta}}{2(2\hat{\Delta}-d+3)}\hat{f}_{\hat{\Delta}+1}(\xi)$$

Bulk blocks

$$G^{\phi_1\bar{\phi}_2}_{\Delta}(\xi) = g^{\Delta_{12}}_{\Delta}(\xi) + \frac{(\Delta-\Delta_{12})(\Delta+\Delta_{12})}{(2\Delta-d+2)(2\Delta-d+4)} g^{\Delta_{12}}_{\Delta+2}(\xi)$$

$$G^{\phi_1\phi_2}_{\Delta}(\xi) = g^{\Delta_{12}}_{\Delta}(\xi)$$

Armed with these blocks, we can start bootstrapping in the ε-expansion!

Future Directions

- Extension to line defects in 3d N=2, and their analytic continuation to d =4
- Combine with bootstrapping theories on the defect
- Multiple defects/fusion of defects
- Higher-point correlators

$$\langle \phi_1 \phi_2 \hat{O}_3 \rangle, \ \langle \phi_1 \hat{O}_2 \phi_3 \hat{O}_4 \rangle$$

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