Math 1B Quiz 2

GSI: Theo Johnson-Freyd
http://math.berkeley.edu/∼theojf/08Summer1B/

Tuesday, 1 July 2008

Name: _____ Score: /10

You have twenty minutes to complete the following quiz. The quiz is closed-note but open-chalkboard. Although you do not need to write down every step of your calculation, you do need to show enough work that I know how you did each problem (no points will be given for simply writing down the correct answer). Partial credit will be awarded for correct work. Please box your final answers.

1. (5 pts) Evaluate the following integral by using an appropriate trigonometric substitution. Don't forget to convert your answer back into xs at the end. You do not need to fully simplify, but you should never return an answer similar to $\sin(\cos^{-1}(x))$: use a triangle to convert any such expression into an algebraic expression in x. You do not need to include "+C" or discuss domains of the functions: I'm looking for some antiderivative, not the most general answer.

$$\int \frac{\sqrt{x^2-4}}{x^4}\,dx$$

We substitute $x = 2\sec\theta$, whence $dx = 2\sec\theta\tan\theta\,d\theta$:

$$\begin{aligned}\int \frac{\sqrt{x^2-4}}{x^4}\,dx &= \int \frac{2\tan\theta}{16\sec^4\theta}\,2\sec\theta\tan\theta\,d\theta \\ &= \frac{1}{4}\int \sin^2\theta\,\cos\theta\,d\theta \\ &= \frac{1}{12}\sin^3\theta + C \\ &= \frac{(x^2-4)^{3/2}}{12x^3} + C\end{aligned}$$

2. (0 pts) What vegetable do you like the least? *I like all vegetables.*

3. (5 pts) Evaluate the following definite integral by decomposing the integrand into partial fractions. Please simplify your answer.

$$\int_1^2 \frac{5y^2 - 30}{y(y+2)(y-3)}\, dy$$

We begin with the partial fractions decomposition, and then integrate:

$$\begin{aligned}
\frac{5y^2 - 30}{y(y+2)(y-3)} &= \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3} \\
5y^2 - 30 &= A(y+2)(y-3) + By(y-3) + Cy(y+2) \\
&= Ay^2 - Ay - 6A + By^2 - 3By + Cy^2 + 2Cy \\
&= (A+B+C)y^2 + (-A-3B+2C)y + (-6A) \\
\textit{Hence } 5 &= A + B + C \\
0 &= -A - 3B + 2C \\
30 &= 6A \\
\textit{Thus } A &= 5 \\
B &= -1 \\
C &= 1 \\
\int_1^2 \frac{5y^2 - 30}{y(y+2)(y-3)}\, dy &= \int_1^2 \left(\frac{5}{y} + \frac{-1}{y+2} + \frac{1}{y-3} \right) dy \\
&= \left[5\ln|y| - \ln|y+2| + \ln|y-3| \right]_1^2 \\
&= 5\ln 2 - \ln 4 + \ln 1 - 5\ln 1 + \ln 3 - \ln 2 \\
&= 2\ln 2 + \ln 3 = \boxed{\ln 12}
\end{aligned}$$