Math 2505: Introductory Analysis

Final Exam - Sample

20 April 2026

Your name:

Exam structure

There are eight questions on this exam, each worth six points.

University academic honour statement:

Dalhousie University has adopted the following statement, based on "The Fundamental Values of Academic Integrity" developed by the International Center for Academic Integrity (ICAI):

> Academic integrity is a commitment to the values of learning in an academic environment. These values include honesty, trust, fairness, responsibility, and respect. All members of the Dalhousie community must acknowledge that academic integrity is fundamental to the value and credibility of academic work and inquiry. We must seek to uphold academic integrity through our actions and behaviours in all our learning environments, our research, and our service.

1. For each of the following statements, fill in the blank with either:

   A: "always" or "all" or "any"

   S: "sometimes but sometimes not" or "some but not some others"

   N: "never" or "none" or "no"

   - Removing finitely many points from a set A _______ changes the set of cluster points of A.
   - For _______ bounded monotone sequences of integers, there is a cut-off after which the sequence remains constant.
   - If $A \subset \mathbb{R}$ has a cluster point c such that $c \in A$, then _______ functions $A \to \mathbb{R}$ are continuous.
   - A bounded continuous function on an open interval _______ has image an interval (possibly open, closed, or half-open).
   - For a monotone function f on a closed interval $[a, b]$, the one-sided limit $\lim_{x \nearrow c} f(x)$ _______ exists.
   - A uniformly convergent sequence of discontinuous functions is _______ discontinuous.

2. Suppose that if $\{x_n\}$ is a sequence for which both $\limsup x_n$ and $\liminf x_n$ exist. Prove that $\{x_n\}$ is bounded.

3. Use mathematical induction to prove that, if S is a set with exactly n elements, then the power set $\mathcal{P}(S)$ of all subsets of S has exactly 2^n elements.

4. Prove that in any ordered field, there are no solutions to the equation $x^2 = -1$.

5. Prove that if $r \in \mathbb{R}$, then $r = \sup\{q \in \mathbb{Q} \text{ s.t. } q < r\}$.

6. Determine whether each of the following sequences of real numbers converges or diverges. For the convergent ones, find their limits. You should show your work but you do not need to write a careful proof.

   - $\cos(\tan n)\sin(1/n)$

   - $\sqrt{n^2+1}-n$

   - $e^{(-1/2)^n}$

7. The *ceiling* $\lceil x \rceil$ of a real number x is the smallest integer n such that $n \geq x$.

   - Explain why there is such an integer, i.e. explain why $\lceil x \rceil$ is a well-defined function $\mathbb{R} \to \mathbb{R}$.

   - At which points $c \in \mathbb{R}$ is $\lceil x \rceil$ continuous?

   - At which points $c \in \mathbb{R}$ is $(\sin x)\lceil(\sin x)\rceil$ continuous?

8. Determine, with proof, whether the sequence of functions

$$f_n(x) = nx \sin(1/xn) : \mathbb{R} \to \mathbb{R}$$

converges uniformly.