MATH 2025: Introductory Analysis

Assignment 1

due January 26

Homework should be submitted *on paper* either in class or to the box by the instructor's office. Proofs do not need to be — and usually should not be — formatted like in a high school logic or geometry class. Rather, they should be formatted as clear mathematical writing: short, complete sentences assembled into paragraphs. Don't forget to "sign post": be clear about what you are proving, what still needs to be proved, what has been proved, and the like.

You are encouraged to work with your classmates, but your writing should be your own. If you do work with other people, please acknowledge (by name) whom you worked with. The purpose of homework assignments is to learn.

Exercises are primarily drawn from *Introduction to Real Analysis* by R.G. Bartle and D.R. Sherbert.

1. Prove that a set T_1 is finite if and only if there is a bijection between T_1 and some finite set T_2. *Hint:* The problem asks you to carefully state the definition of "finite set" and then explain both directions of the implication. In one direction, you know that T_1 is finite and you need to produce a T_2 which is also finite and a bijection $T_1 \cong T_2$. In the other direction, you know that you have a bijection $T_1 \cong T_2$ and that T_2 is finite and you need to prove that T_1 is finite.

2. Let $S := \{1, 2\}$ and $T := \{\text{moose}, \text{puffin}, \text{beaver}\}$.

   (a) How many injections are there from S to T?

   (b) How many surjections are there from S to T?

   (c) How many injections are there from T to S?

   (d) How many surjections are there from T to S?

   You should justify your answers, but you do not need to write your justification as a formal proof.

3. Exhibit a bijection between $\mathbb{N}$ and the set of all odd integers strictly greater than 13. *Hint:* The problem asks you both to give the bijection and also, implicitly, to explain why it is a bijection. You may structure your explanation as a formal proof but you may also structure your explanation as informal text.

4. * Prove that the collection $\mathcal{F}(\mathbb{N})$ of all *finite* subsets of $\mathbb{N}$ is countable.

5. Prove that if $a, b \in \mathbb{R}$, then:

   (a) $-(a + b) = (-a) + (-b)$

   (b) $(-a)(-b) = ab$

(c) $-(a/b) = (-a)/b = a/(-b)$ if $b \neq 0$.

6. Prove that if $a \in \mathbb{R}$ satisfies $a^2 = a$, then $a = 0$ or $a = 1$.

7. (a) Give an example of real numbers a, b, c, d with $0 < a < b$ and $c < d < 0$ such that $ac < bd$.

   (b) Give an example of real numbers a, b, c, d with $0 < a < b$ and $c < d < 0$ such that $ac > bd$.

8. Prove that $|a| = \sqrt{a^2}$.

9. Prove that $|a + b| = |a| + |b|$ if and only if $ab \geq 0$.