MATH 2505: Intro to Real Analysis

07. Jan. 2026

Lecture 1: Review of Set Theory

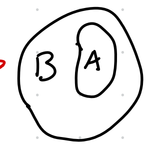
Definition:

- For this course "naive set theory": a set is a collection of elements
- Two sets are the same when their elements are the same

Notation:

$a \in A \longrightarrow$ "element a is in A"

$A \subset B$ or $A \subseteq B \longrightarrow$ "every element of A is also an element of B" $\longrightarrow$



$A \subsetneq B$ $\longrightarrow$ the sets are not equal

Some important sets

$\mathbb{R} \longrightarrow$ real numbers $\quad \mathbb{Z} \longrightarrow$ integers $\quad \mathbb{Q} \longrightarrow$ quotients of integers (rational #s)

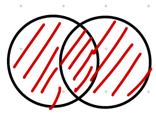
$\mathbb{N} \longrightarrow$ natural numbers $= \{0, 1, 2, \ldots\}$ $\quad \emptyset \longrightarrow$ empty set (no elements) $\longrightarrow$ the set s.t. $\forall a,\ a \notin \emptyset$

$\mathbb{Z}^+ \longrightarrow$ strictly positive $(\mathbb{Z}_{>0})$ $\quad$ or $\quad \mathbb{N}_{<7} \longrightarrow$ natural numbers less than 7

More on Sets

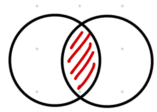
Union

$A \cup B$



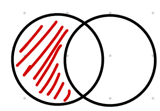
intersection

$A \cap B$



set difference

$A \setminus B$



more generally given an indexed family of sets e.g. a list $A_0, A_1, A_2, \ldots$

$\bigcup_{i=0}^{\infty} A_i \qquad \bigcap_{i=0}^{\infty} A_i$

$a \in \bigcap_{i=0}^{\infty} A_i \iff a \in A_i$ for every i $\quad$ "if and only if"

$a \in \bigcup_{i=0}^{\infty} A_i \iff a \in A_i$ for some i

Example: $A_i = \left(\frac{-1}{i+1}, \frac{1}{i+1}\right) \subset \mathbb{R}$ $\qquad \bigcap_{i=0}^{\infty} \left(-\frac{1}{i+1}, \frac{1}{i+1}\right) = \{0\}$ $\qquad$ or for union $\bigcup_{i=0}^{\infty} (-i, i) = \mathbb{R}$

Functions

- Suppose A & B are sets; the product $A \times B$ is a set of ordered pairs (a,b) s.t. $a \in A$ & $b \in B$
  Its all the possible pairs (a,b)

Define: A subset $\gamma \subset A \times B$ is a graph of a function if $\forall a \in A \ \exists! \ (a,b) \in \gamma$

"for every" $\quad$ "there exists" $\quad$ "unique"

Spelled out: given a, look at $\gamma_a := \{(a',b') \text{ s.t. } a' = a\}$

γ is a function if, γ_a has exactly one element $\forall a \in A$

$f: A \to B$ if f is a function

$f(a) \in B$ will be the second coordinate of the unique element in γ_a

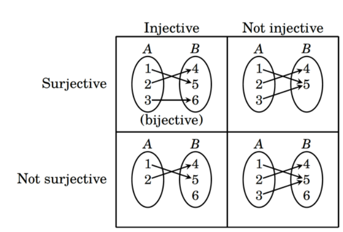
Definitions:

$f: A \to B \longrightarrow$ injective if $\forall b \in B$ there is at most one a s.t. $f(a) = b$

$f: A \to B \longrightarrow$ surjective if $\forall b \in B$ there is at least one a s.t. $f(a) = b$

A is the domain

B is the codomain



Function Composition:

$f: A \to B$; $g: B \to C$ → composing the two we get $gf = g \circ f : A \to C$

$a \mapsto g(f(a))$ → means a specific input "a" is taken to $g(f(a))$

In terms of graphs we write $gf \subset A \times C$

$= \{(a,c) \mid \exists\, b \in B \text{ s.t. } (a,b) \in \gamma_f \text{ and } (b,c) \in \gamma_g\}$

The identity function

→ For every set A $e_A : A \to A$ defined by γ_{e_A} is the diagonal in $A \times A$ → $\{(a,a) \mid a \in A\}$

if $f: A \to B$ $f \circ e_A = f$

$e_B \circ f = f$

→ if $f: A \to B$ is bijective the define inverse $f^{-1}: B \to A$ to be the function that sends $b \in B$ to the unique $a \in A$ s.t. $f(a) = b$

$\gamma_{f^{-1}}$ is the reflection of γ_f then $f^{-1} \circ f = e_A$; $f \circ f^{-1} = e_B$