Math 2505: Introductory Analysis

Midterm 1

2 February 2026

Your name:

Exam structure

There are four questions on this exam, each worth six points.

University academic honour statement:

Dalhousie University has adopted the following statement, based on "The Fundamental Values of Academic Integrity" developed by the International Center for Academic Integrity (ICAI):

> Academic integrity is a commitment to the values of learning in an academic environment. These values include honesty, trust, fairness, responsibility, and respect. All members of the Dalhousie community must acknowledge that academic integrity is fundamental to the value and credibility of academic work and inquiry. We must seek to uphold academic integrity through our actions and behaviours in all our learning environments, our research, and our service.

Part A.

1. For each of the following statements, fill in the blank with either "always" (A), "sometimes but sometimes not" (S), or "never" (N). You do not need to justify your answers.

- A bounded nonempty subset of $\mathbb{N}$ _____ has a greatest element.
- A bounded nonempty subset of $\mathbb{R}$ _____ has a greatest element.
- The composition of two surjections is _____ an injection.
- Given negative real numbers $a, b \in \mathbb{R}_{<0}$, their arithmetic mean $\frac{a+b}{2}$ is _____ strictly less than their geometric mean $\sqrt{ab}$.
- In a field, you can _____ cancel additions: $a + x = b + x$ implies $a = b$.
- In a field, you can _____ cancel multiplication: $a \cdot x = b \cdot x$ implies $a = b$.

2. Define the following notions:

- Absolute value of a real number.

- Inverse of a bijective function.

- Commutativity of multiplication.

3. For each of the following sets, find its inf and sup or show that they do not exist:

- $\{x \in \mathbb{N} \text{ s.t. } 2^x > x + 1\}$.

- $\{x \in \mathbb{Q} \text{ s.t. } x^2 \leq 3\}$.

4. Prove that in an ordered field, $x < y$ then $x^n < y^n$ for all odd positive integers n.