Math 2505: Introductory Analysis

Midterm 1

2 February 2026

Your name:

Exam structure

There are four questions on this exam, each worth six points.

University academic honour statement:

Dalhousie University has adopted the following statement, based on "The Fundamental Values of Academic Integrity" developed by the International Center for Academic Integrity (ICAI):

> Academic integrity is a commitment to the values of learning in an academic environment. These values include honesty, trust, fairness, responsibility, and respect. All members of the Dalhousie community must acknowledge that academic integrity is fundamental to the value and credibility of academic work and inquiry. We must seek to uphold academic integrity through our actions and behaviours in all our learning environments, our research, and our service.

1. For each of the following statements, fill in the blank with either "always" (A), "sometimes but sometimes not" (S), or "never" (N). You do not need to justify your answers.

   - A nonempty subset of $\mathbb{N}$ _______ has a greatest element.
   - The composition of two surjections is _______ a surjection.
   - Given positive real numbers $a, b \in \mathbb{R}_{>0}$, their arithmetic mean $\frac{a+b}{2}$ is _______ strictly less than their geometric mean $\sqrt{ab}$.
   - In a field, the product of two nonzero elements is _______ nonzero.
   - In a field, the sum of two nonzero elements is _______ nonzero.
   - In an ordered field, the product of two strictly negative elements is _______ strictly negative.

2. • State the *Pigeonhole Principle*.

• Explain why the Pigeonhole Principle implies that if S is a finite set, then there is a unique natural number n with $|S| = n$. In particular, explain that "$|S| = n$" is *defined* in terms of the existence of a certain bijection, and explain why the Pigeonhole Principle implies that if $|S| = n$ and $|S| = m$, then $n = m$.

3. Write the following sets as unions of intervals. You should show your work but you do not need to write careful proofs.

- $\{x \in \mathbb{R} \text{ s.t. } 3 \leq |x+2| \leq 5\}$

- $\{x \in \mathbb{R} \text{ s.t. } x^3 < x^2\}$.

- $\{x \in \mathbb{R} \text{ s.t. } x < \frac{1}{n} \text{ for all } n \in \mathbb{Z}_{>0}\}$

4. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function such that if $x < y$ then $f(x) < f(y)$. Show that if $S \subset \mathbb{R}$ is a bounded set, then $f(S) := \{f(x) \text{ s.t. } x \in S\}$ is bounded and that $\sup(f(S)) \leq f(\sup S)$.