Math 2505: Introductory Analysis

Midterm 2

27 March 2026

Your name:

Exam structure

There are four questions on this exam, each worth six points.

University academic honour statement:

Dalhousie University has adopted the following statement, based on "The Fundamental Values of Academic Integrity" developed by the International Center for Academic Integrity (ICAI):

> Academic integrity is a commitment to the values of learning in an academic environment. These values include honesty, trust, fairness, responsibility, and respect. All members of the Dalhousie community must acknowledge that academic integrity is fundamental to the value and credibility of academic work and inquiry. We must seek to uphold academic integrity through our actions and behaviours in all our learning environments, our research, and our service.

1. For each of the following statements, fill in the blank with either "always" (A), "sometimes but sometimes not" (S), or "never" (N). You do not need to justify your answers. Read carefully.

   - A bounded sequence of real numbers _______ converges.
   - A sequence of real numbers _______ contains a monotonic subsequence.
   - The absolute value of a convergent sequence _______ converges.
   - A convergent sequence and a convergent subsequence of it _______ have different limits.
   - Any cluster point c of a set $A \subset \mathbb{R}$ _______ has a neighbourhood $V_\delta(c)$ such that $V_\delta(c) \cap A$ is finite.
   - A function $f : \mathbb{R} \to \mathbb{R}$ which converges at some point c is _______ bounded on some neighbourhood $V_\delta(c)$.

2. Show that each of the following sequences diverges:

- $x_n = (-1)^n + n^2$

- $x_n = \cos(n^2\pi)$

- $x_n = \dfrac{1}{\sqrt{n+1} - \sqrt{n}}$

3. Suppose that x_n and y_n are sequences of real numbers such that $y_n \leq x_n$ for all n.

- Prove that if x_n is bounded above, then y_n is bounded above. Explain why is this equivalent to the following statement: "If $\limsup x_n$ exists then $\limsup y_n$ exists."

- Prove that $\limsup y_n \leq \limsup x_n$.

- Give an example with $y_n < x_n$ for all n, but with $\limsup y_n = \limsup x_n$.

4. Prove that if x_n is a convergent sequence, then $\lim x_n$ is a cluster point of the set $\{x_n : n \in \mathbb{N}\}$.