Recent progress on the classification of fusion higher categories

SCGCS status report and outlook

Theo Johnson-Freyd, 23 Feb 2024

My team

Postdocs

- Lukas Müller
  - ↪ non s.s. modular functors
  - ↪ non s.s. string nets
  - ↪ higher geometry
  - ↪ higher skein theory
  - ↪ ...
- Luuk Stehouwer
  - ↪ higher dagger categories
  - ↪ categorical spin-statistics
  - ↪ ...

Graduate Students

- Adrien DeLazzer Meunier
  - ↪ nonminimal nondeg extensions
- Daniel Teixeira
  - ↪ models of higher cats
- Ruizhi Liu
  - ↪ anomalies in cond-mat

Classification of (multi)fusion n-categories

<u>n=0:</u> mFOCs = f.d. semisimple algebras.

Artin-Wedderburn theorem: every fd ss alg is

$$\bigoplus \mathrm{Mat}_n(\text{division ring}).$$

These are classifiable in terms of Galois cohomology

<u>n=1:</u> Totally wild. Even if you knew all of homotopy theory, you'd have no idea. Ask Julia.

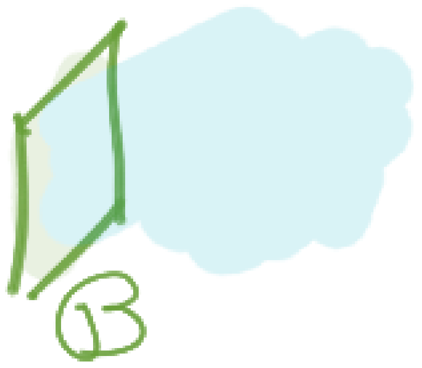
<u>n=2:</u> New. Joint with Décoppet*, Huston, Nikshych, Penneys, Plavnik*, Reutter, and Yu.

* SCGCS members.

Construction of mF2Cs via gauging

mFnC $\equiv$ n+1D quiche (n+2D symTFT).

Pick up a braided fusion 1-category $\mathbb{B}$. It determines a 3D quiche



← bulk is 4D $\underbrace{CY(\mathbb{B})}_{\text{determined by } \mathbb{B} \text{ up to Witt equiv.}}$.

Fact about BF1Cs: Every BF1C is canonically

$$\mathbb{B} \simeq (\mathbb{B}')^G$$

the fixed points of an action $G \circlearrowright \mathbb{B}'$, with G a finite gp and $\mathbb{B}'$ either nondegenerate or slightly degenerate.

$CY(\mathbb{B})$ is invertible.

slightly degenerate: ⟶ $CY(\mathbb{B}) \simeq$ invertible $\times$ (dynamical spin str)

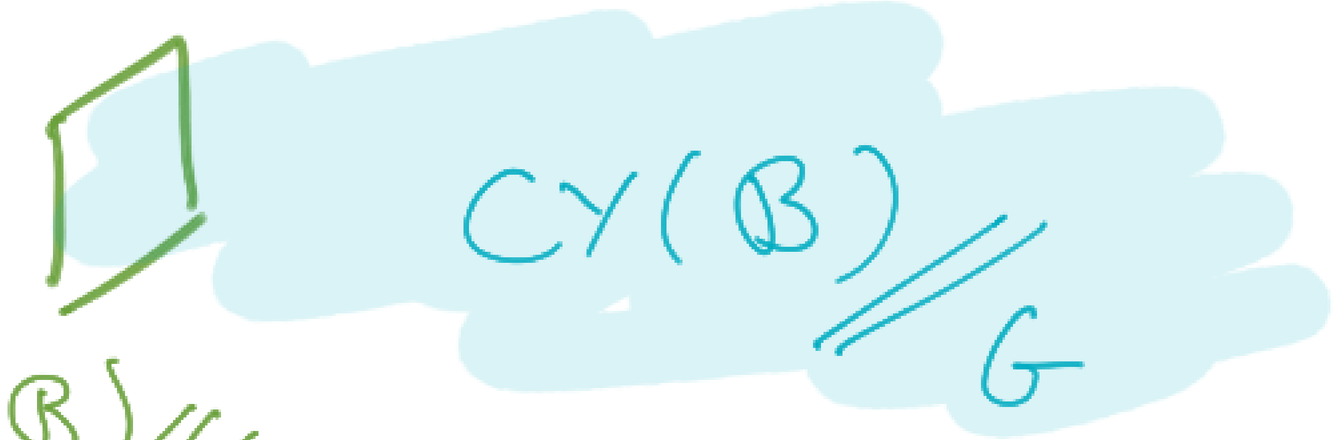
Construction of mF2Cs via gauging

So let's restrict attention to $\mathbb{B}$ nondeg or slightly deg.

Pick a finite gp G and a (nonanomalous) action on $CY(\mathbb{B})$.

Pick a subgp $H \subseteq G$ and an extension of the H-action to the boundary. H = spontaneously unbroken symmetries.

These data can be gauged:



$\mathbb{B}/\!/_H = (\mathrm{Ind}_H^G \mathbb{B})/\!/G$

Main Theorem

Every F2C arises canonically from this construction:

there is an equivalence of 3-groupoids

$$\{\text{F2Cs}\} \cong \{\text{tuples } (G,\ H \leq G,\ \mathcal{B},\ H \curvearrowright \mathcal{B},\ G \curvearrowright \mathcal{Y}(\mathcal{B}))\}$$

finite gp (or id)

nondeg or slightly deg

No inherent noninvertibility in dimension 2.

using corresponding hard results for BF1Cs

Corollaries:

- rank-finiteness for F2Cs
- Ocneanu rigidity for F2Cs

- Hyper gp (fusion rules) $= {}_H\backslash^{G}/_{H}$

Fusion (>2)-categories

"nondeg v.s. slightly deg" $\rightsquigarrow$ more complicated, but we basically understand it.

"nondeg BF1C" $\rightsquigarrow$ simpler. only see "quadratic" $\int A\,dA$ type theories.

Details depend on dimension mod 2.

Only inherent noninvertibility is F1Cs and BF1Cs.

The rest is homotopy theory, L-thy, Galois thy, ...