Luminous

I woke, disorientated, unsure why. I knew I was lying on the narrow, lumpy single bed in Room 22 of the Hotel Fleapit; after almost a month in Shanghai, the topography of the mattress was depressingly familiar. But there was something wrong with the way I was lying; every muscle in my neck and shoulders was protesting that nobody could end up in this position from natural causes, however badly he'd slept.

And I could smell blood.

I opened my eyes. A woman I'd never seen before was kneeling over me, slicing into my left tricep with a disposable scalpel. I was lying on my side, facing the wall, one hand and one ankle cuffed to the head and foot of the bed.

Something cut short the surge of visceral panic before I could start stupidly thrashing about, instinctively trying to break free. Maybe an even more ancient response – catatonia in the face of danger – took on the adrenaline and won. Or maybe I just decided that I had no right to panic when I'd been expecting something like this for weeks.

I spoke softly, in English. 'What you're in the process of hacking out of me is a necrotrap. One heartbeat without oxygenated blood, and the cargo gets fried.'

My amateur surgeon was compact, muscular, with short black hair. Not Chinese; Indonesian, maybe. If she was surprised that I'd woken prematurely, she didn't show it. The gene-tailored hepatocytes I'd acquired in Hanoi could degrade almost anything from morphine to curare; it was a good thing the local anaesthetic was beyond their reach.

Without taking her eyes off her work, she said, 'Look on the table next to the bed.'

I twisted my head around. She'd set up a loop of plastic tubing full of blood – mine, presumably – circulated and aerated by a small pump. The stem of a large funnel fed into the loop, the intersection controlled by a valve of some kind. Wires trailed from the pump to a sensor taped to the inside of my elbow, synchronising the artificial pulse with the real. I had no doubt that she could tear the trap from my vein and insert it into this substitute without missing a beat.

I cleared my throat and swallowed. 'Not good enough. The trap knows my blood-pressure profile exactly. A generic heartbeat won't fool it.'

'You're bluffing.' But she hesitated, scalpel raised. The hand-held MRI scanner she'd used to find the trap would have revealed its basic configuration, but few fine details of the engineering – and nothing at all about the software.

'I'm telling you the truth.' I looked her squarely in the eye, which wasn't easy given our awkward geometry. 'It's new, it's Swedish. You anchor it in a vein forty-eight hours in advance, put yourself through a range of typical activities so it can memorise the rhythms . . . then you inject the cargo into the trap. Simple, foolproof, effective.' Blood trickled down across my chest onto the sheet. I was suddenly very glad that I hadn't buried the thing deeper, after all.

'So how do you retrieve the cargo, yourself?'

'That would be telling.'

'Then tell me now, and save yourself some trouble.' She rotated the scalpel between thumb and forefinger impatiently. My skin did a cold burn all over, nerve ends jangling, capillaries closing down as blood dived for cover.

I said, '*Trouble* gives me hypertension.'

She smiled down at me thinly, conceding the stalemate, then peeled off one stained surgical glove, took out her notepad, and made a call to a medical-equipment supplier. She listed some devices which would get around the problem – a blood-pressure probe, a more sophisticated pump, a suitable computerised inter-

Two blocks from the hotel I stopped dead, my legs almost giving way beneath me. It wasn't just shock – a delayed reaction, a belated acceptance of how close I'd come to being slaughtered. The burglar's clinical violence was chilling enough, but what it implied was infinitely more disturbing.

Industrial Algebra were paying big money, violating international law, taking serious risks with their corporate and personal futures. The arcane abstraction of the defect was being dragged into the world of blood and dust, boardrooms and assassins, power and pragmatism.

And the closest thing to certainty humanity had ever known was in danger of dissolving into quicksand.

* * *

It had all started out as a joke. Argument for argument's sake. Alison and her infuriating heresies.

'A mathematical theorem,' she'd proclaimed, 'only becomes true when a physical system tests it out: when the system's behaviour depends in some way on the theorem being *true* or *false*.'

It was June 1994. We were sitting in a small paved courtyard, having just emerged, yawning and blinking, into the winter sunlight from the final lecture in a one-semester course on the philosophy of mathematics – a bit of light relief from the hard grind of the real stuff. We had fifteen minutes to kill before meeting some friends for lunch. It was a social conversation – verging on mild flirtation – nothing more. Maybe there were demented academics, lurking in dark crypts somewhere, who held views on the nature of mathematical truth which they were willing to die for. But we were twenty years old, and we *knew* it was all angels on the head of a pin.

I said, 'Physical systems don't create mathematics. Nothing *creates* mathematics – it's timeless. All of number theory would still be exactly the same, even if the universe contained nothing but a single electron.'

Alison snorted. 'Yes, because even *one electron*, plus a space-time to put it in, needs all of quantum mechanics and all of general relativity – and all the mathematical infrastructure they entail. One

particle floating in a quantum vacuum needs half the major results of group theory, functional analysis, differential geometry—'

'OK, OK! I get the point. But if that's the case . . . the events in the first picosecond after the Big Bang would have "constructed" every last mathematical truth required by *any* physical system, all the way to the Big Crunch. Once you've got the mathematics which underpins the Theory of Everything . . . that's it, that's all you ever need. End of story.'

'But it's not. To *apply* the Theory of Everything to a particular system, you still need all the mathematics for dealing with *that system* – which could include results far beyond the mathematics which the TOE itself requires. I mean, fifteen billion years after the Big Bang, someone can still come along and prove, say . . . Fermat's Last Theorem.' Andrew Wiles at Princeton had recently announced a proof of the famous conjecture, although his work was still being scrutinised by his colleagues, and the final verdict wasn't yet in. 'Physics never needed *that* before.'

I protested, 'What do you mean, "before"? Fermat's Last Theorem never has – and never will – have anything to do with any branch of physics.'

Alison smiled sneakily. 'No *branch*, no. But only because the class of physical systems whose behaviour depends on it is so ludicrously specific: the brains of mathematicians who are trying to validate the Wiles proof.

'Think about it. Once you start trying to prove a theorem, then even if the mathematics is so "pure" that it has no relevance to any other object in the universe . . . you've just made it relevant to *yourself*. You have to choose *some* physical process to test the theorem – whether you use a computer, or a pen and paper . . . or just close your eyes and shuffle *neurotransmitters*. There's no such thing as a proof which doesn't rely on physical events, and whether they're inside or outside your skull doesn't make them any less real.'

'Fair enough,' I conceded warily. 'But that doesn't mean—'

'And maybe Andrew Wiles's brain – and body, and notepaper – comprised the first physical system whose behaviour depended on

the theorem being true or false. But I don't think human actions have any special role . . . and if some swarm of quarks had done the same thing blindly, fifteen billion years before – executed some purely random interaction which just happened to test the conjecture in some way – then *those quarks* would have constructed FLT long before Wiles. We'll never know.'

I opened my mouth to complain that no swarm of quarks could have tested the infinite number of cases encompassed by the theorem, but I caught myself just in time. That was true, but it hadn't stopped Wiles. A finite sequence of logical steps linked the axioms of number theory – which included some simple generalities about *all* numbers – to Fermat's own sweeping assertion. And if a mathematician could test those logical steps by manipulating a finite number of physical objects for a finite amount of time – whether they were pencil marks on paper, or neurotransmitters in his or her brain – then all kinds of physical systems could, in theory, mimic the structure of the proof . . . with or without any awareness of what it was they were 'proving'.

I leant back on the bench and mimed tearing out hair. 'If I wasn't a die-hard Platonist before, you're forcing me into it! Fermat's Last Theorem didn't *need* to be proved by anyone, or stumbled on by any random swarm of quarks. If it's true, it was always true. Everything implied by a given set of axioms is logically connected to them, timelessly, eternally . . . even if the links couldn't be traced by people – or quarks – in the lifetime of the universe.'

Alison was having none of this; every mention of *timeless and eternal truths* brought a faint smile to the corners of her mouth, as if I was affirming my belief in Santa Claus. She said, 'So who, or what, pushed the consequences of "There exists an entity called zero" and "Every X has a successor", et cetera, all the way to FLT and beyond, before the universe had a chance to test out any of it?'

I stood my ground. 'What's joined by logic is just . . . *joined*. Nothing has to happen: consequences don't have to be "pushed" into existence by anyone, or anything. Or do you imagine that the first events after the Big Bang, the first wild jitters of the quark-gluon plasma, stopped to fill in all the logical gaps? You think the quarks

reasoned: Well, so far we've done A and B and C, but now we mustn't do D, because D would be logically inconsistent with the other mathematics we've "invented" so far . . . even if it would take a five-hundred-thousand-page proof to spell out the inconsistency?'

Alison thought it over. 'No. But what if event D took place, regardless? What if the mathematics it implied *was* logically inconsistent with the rest, but it went ahead and happened anyway . . . because the universe was too young to have computed the fact that there was any discrepancy?'

I must have sat and stared at her, open-mouthed, for about ten seconds. Given the orthodoxies we'd spent the last two-and-a-half years absorbing, this was a seriously outrageous statement.

'You're claiming that . . . *mathematics* might be strewn with primordial defects in consistency? Like space might be strewn with cosmic strings?'

'Exactly.' She stared back at me, feigning nonchalance. 'If space-time doesn't join up with itself smoothly, everywhere, why should mathematical logic?'

I almost choked. 'Where do I begin? What happens – now – when some physical system tries to link theorems across the defect? If theorem D has been rendered "true" by some over-eager quarks, what happens when we program a computer to disprove it? When the software goes through all the logical steps which link A, B and C – which the quarks have also made true – to the contradiction, the dreaded not-D, does it succeed or doesn't it?'

Alison side-stepped the question. 'Suppose they're both true: D and not-D. Sounds like the end of mathematics, doesn't it? The whole system falls apart, instantly. From D and not-D together you can prove anything you like: one equals zero, day equals night. But that's just the boring-old-fart Platonist view, where logic travels faster than light, and computation takes no time at all. People live with omega-inconsistent theories, don't they?'

Omega-inconsistent number theories were non-standard versions of arithmetic, based on axioms which 'almost' contradicted each other – their saving grace being that the contradictions could only show up in 'infinitely long proofs' (which were formally

disallowed, quite apart from being physically impossible). That was perfectly respectable modern mathematics, but Alison seemed prepared to replace 'infinitely long' with just plain 'long', as if the difference hardly mattered, in practice.

I said, 'Let me get this straight. What you're talking about is taking ordinary arithmetic – no weird counter-intuitive axioms, just the stuff every ten-year-old *knows* is true – and proving that it's inconsistent, in a finite number of steps?'

She nodded blithely. 'Finite, but large. So the contradiction would rarely have any physical manifestation – it would be "computationally distant" from everyday calculations, and everyday physical events. I mean . . . one cosmic string, somewhere out there, doesn't destroy the universe, does it? It does no harm to anyone.'

I laughed drily. 'So long as you don't get too close. So long as you don't tow it back to the solar system and let it twitch around slicing up planets.'

'Exactly.'

I glanced at my watch. 'Time to come down to Earth, I think. You know we're meeting Julia and Ramesh—?'

Alison sighed theatrically. 'I know, I know. And this would bore them witless, poor things – so the subject's closed, I promise.' She added wickedly, 'Humanities students are so *myopic*.'

We set off across the tranquil leafy campus. Alison kept her word, and we walked in silence; carrying on the argument up to the last minute would have made it even harder to avoid the topic once we were in polite company.

Halfway to the cafeteria, though, I couldn't help myself.

'If someone ever *did* program a computer to follow a chain of inferences across the defect, what do you claim would actually happen? When the end result of all those simple, trustworthy logical steps finally popped up on the screen, which group of primordial quarks would win the battle? And please don't tell me that the whole computer just conveniently vanishes.'

Alison smiled, tongue-in-cheek at last. 'Get real, Bruno. How can you expect me to answer that, when the mathematics needed to predict the result doesn't even *exist* yet? Nothing I could say would

be true or false – until someone's gone ahead and done the experiment.'

* * *

I spent most of the day trying to convince myself that I wasn't being followed by some accomplice (or rival) of the surgeon, who might have been lurking outside the hotel. There was something disturbingly Kafkaesque about trying to lose a tail who might or might not have been real: no particular face I could search for in the crowd, just the abstract idea of a pursuer. It was too late to think about plastic surgery to make me look Han Chinese – Alison had raised this as a serious suggestion, back in Vietnam – but Shanghai had over a million foreign residents, so with care even an Anglophone of Italian descent should have been able to vanish.

Whether or not I was up to the task was another matter.

I tried joining the ant-trails of the tourists, following the path of least resistance from the insane crush of the Yuyuan Bazaar (where racks bursting with ten-cent watch-PCs, mood-sensitive contact lenses, and the latest karaoke vocal implants, sat beside bamboo cages of live ducks and pigeons) to the one-time residence of Sun Yatsen (whose personality cult was currently undergoing a mini-series-led revival on Phoenix TV, advertised on ten thousand buses and ten times as many T-shirts). From the tomb of the writer Lu Xun ('Always think and study . . . visit the generals, then visit the victims; see the realities of your time with open eyes' – no prime time for *him*) to the Hongkou McDonalds (where they were giving away small plastic Andy Warhol figurines, for reasons I couldn't fathom).

I mimed leisurely window-shopping between the shrines, but kept my body language sufficiently unfriendly to deter even the loneliest Westerner from attempting to strike up a conversation. If foreigners were unremarkable in most of the city, they were positively eye-glazing here – even to each other – and I did my best to offer no one the slightest reason to remember me.

Along the way I checked for messages from Alison, but there were none. I left five of my own, tiny abstract chalk marks on bus shelters

and park benches – all slightly different, but all saying the same thing: CLOSE BRUSH, BUT SAFE NOW. MOVING ON.

By early evening, I'd done all I could to throw off my hypothetical shadow, so I headed for the next hotel on our agreed but unwritten list. The last time we'd met face-to-face, in Hanoi, I'd mocked all of Alison's elaborate preparations. Now I was beginning to wish that I'd begged her to extend our secret language to cover more extreme contingencies. FATALLY WOUNDED. BETRAYED YOU UNDER TORTURE. REALITY DECAYING. OTHERWISE FINE.

The hotel on Huaihai Zhonglu was a step up from the last one, but not quite classy enough to refuse payment in cash. The desk clerk made polite small-talk, and I lied as smoothly as I could about my plans to spend a week sightseeing before heading for Beijing. The bellperson smirked when I tipped him too much – and I sat on my bed for five minutes afterwards, wondering what significance to read into *that*.

I struggled to regain a sense of proportion.

Industrial Algebra *could* have bribed every single hotel employee in Shanghai to be on the lookout for us, but that was a bit like saying that, in theory, they could have duplicated our entire twelve-year search for defects, and not bothered to pursue us at all. There was no question that they wanted what we had, badly, but what could they actually do about it? Go to a merchant bank (or the Mafia, or a Triad) for finance? That might have worked if the cargo had been a stray kilogram of plutonium, or a valuable gene sequence, but only a few hundred thousand people on the planet would be capable of understanding what the defect *was*, even in theory. Only a fraction of that number would believe that such a thing could really exist, and even fewer would be both wealthy and immoral enough to invest in the business of exploiting it.

The stakes appeared to be infinitely high, but that didn't make the players omnipotent.

Not yet.

I changed the dressing on my arm, from sock to handkerchief, but the incision was deeper than I'd realised, and it was still bleeding thinly. I left the hotel – and found exactly what I needed in a

twenty-four-hour emporium just ten minutes away. Surgical-grade tissue-repair cream: a mixture of collagen-based adhesive, antiseptic, and growth factors. The emporium wasn't even a pharmaceuticals outlet: it just had aisle after aisle packed with all kinds of unrelated odds and ends, laid out beneath the unblinking blue-white ceiling panels. Canned food, PVC plumbing fixtures, traditional medicines, rat contraceptives, video ROMS. It was a random cornucopia, an almost organic diversity – as if the products had all just grown on the shelves from whatever spores the wind had happened to blow in.

I headed back to the hotel, pushing my way through the relentless crowds, half seduced and half sickened by the odours of cooking, dazed by the endless vista of holograms and neon in a language I barely understood. Fifteen minutes later, reeling from the noise and humidity, I realised that I was lost.

I stopped on a street corner and tried to get my bearings. Shanghai stretched out around me, dense and lavish, sensual and ruthless – a Darwinian economic simulation self-organised to the brink of catastrophe. The Amazon of commerce: this city of sixteen million had more industry of every kind, more exporters and importers, more wholesalers and retailers, traders and re-sellers and recyclers and scavengers, more billionaires and more beggars, than most nations on the planet.

Not to mention more computing power.

China itself was reaching the cusp of its decades-long transition from brutal totalitarian communism to brutal totalitarian capitalism: a slow seamlesss morph from Mao to Pinochet set to the enthusiastic applause of its trading partners and the international financial agencies. There'd been no need for a counter-revolution – just layer after layer of carefully reasoned Newspeak to pave the way from previous doctrine to the stunningly obvious conclusion that private property, a thriving middle class, and a few trillion dollars worth of foreign investment were exactly what the Party had been aiming for all along.

The apparatus of the police state remained as essential as ever. Trade unionists with decadent bourgeois ideas about uncompetitive

wages, journalists with counter-revolutionary notions of exposing corruption and nepotism, and any number of subversive political activists spreading destabilising propaganda about the fantasy of free elections, all needed to be kept in check.

In a way, Luminous was a product of this strange transition from communism to not-communism in a thousand tiny steps. No one else, not even the US defence research establishment, possessed a single machine with so much power. The rest of the world had succumbed long ago to networking, giving up their imposing supercomputers with their difficult architecture and customised chips for a few hundred of the latest mass-produced work stations. In fact, the biggest computing feats of the twenty-first century had all been farmed out over the Internet to thousands of volunteers, to run on their machines whenever the processors would otherwise be idle. That was how Alison and I had mapped the defect in the first place: seven thousand amateur mathematicians had shared the joke, for twelve years.

But now the net was the very opposite of what we needed, and only Luminous could take its place. And though only the People's Republic could have paid for it, and only the People's Institute for Advanced Optical Engineering could have built it . . . only Shanghai's QIPS Corporation could have sold time on it to the world – while it was still being used to model hydrogen-bomb shock waves, pilotless fighter jets, and exotic anti-satellite weapons.

I finally decoded the street signs, and realised what I'd done: I'd turned the wrong way coming out of the emporium, it was as simple as that.

I retraced my steps, and I was soon back on familiar territory.

* * *

When I opened the door of my room, Alison was sitting on the bed.

I said, 'What is it with locks in this city?'

We embraced, briefly. We'd been lovers, once, but that was long over. And we'd been friends for years afterwards, but I wasn't sure if that was still the right word. Our whole relationship now was too functional, too spartan. Everything revolved around the defect, now.

She said, 'I got your message. What happened?'

I described the morning's events.

'You know what you should have done?'

That stung. 'I'm still here, aren't I? The cargo's still safe.'

'You should have killed her, Bruno.'

I laughed. Alison gazed back at me placidly, and I looked away. I didn't know if she was serious – and I didn't much want to find out.

She helped me apply the repair cream. My toxin was no threat to her: we'd both installed exactly the same symbionts, the same genotype from the same unique batch in Hanoi. But it was strange to feel her bare fingers on my broken skin, knowing that no one else on the planet could touch me like this, with impunity.

Ditto for sex, but I didn't want to dwell on that.

As I slipped on my jacket, she said, 'So guess what we're doing at five a.m. tomorrow.'

'Don't tell me: I fly to Helsinki, and you fly to Cape Town? Just to throw them off the scent.'

That got a faint smile. 'Wrong. We're meeting Yuen at the Institute – and spending half an hour on Luminous.'

'*You* are brilliant.' I bent over and kissed her on the forehead. 'But I always knew you'd pull it off.'

And I should have been delirious, but the truth was my guts were churning; I felt almost as trapped as I had upon waking cuffed to the bed. If Luminous had remained beyond our reach (as it should have, since we couldn't afford to hire it for a microsecond at the going rate) we would have had no choice but to destroy all the data, and hope for the best. Industrial Algebra had no doubt dredged up a few thousand fragments of the original Internet calculations, but it was clear that, although they knew exactly what we'd found, they still had no idea where we'd found it. If they'd been forced to start their own random search – constrained by the need for secrecy to their own private hardware – it might have taken them centuries.

There was no question now, though, of backing away and leaving everything to chance. We were going to have to confront the defect in person.

'How much did you have to tell him?'

'Everything.' She walked over to the washbasin, removed her shirt, and began wiping the sweat from her neck and torso with a flannel. 'Short of handing over the map. I showed him the search algorithms and their results, and all the programs we'll need to run on Luminous – all stripped of specific parameter values, but enough for him to validate the techniques. He wanted to see direct evidence of the defect, of course, but I held out on that.'

'And how much did he believe?'

'He's reserved judgement. The deal is, we get half an hour's unimpeded access, but he gets to observe everything we do.'

I nodded, as if my opinion made any difference, as if we had any choice. Yuen Ting-fu had been Alison's supervisor for her PhD on advanced applications of ring theory, when she'd studied at Fu-tan University in the late nineties. Now he was one of the world's leading cryptographers, working as a consultant to the military, the security services, and a dozen international corporations. Alison had once told me that she'd heard he'd found a polynomial-time algorithm for factoring the product of two primes; that had never been officially confirmed, but such was the power of his reputation that almost everyone on the planet had stopped using the old RSA encryption method as the rumour had spread. No doubt time on Luminous was his for the asking, but that didn't mean he couldn't still be imprisoned for twenty years for giving it away to the wrong people, for the wrong reasons.

I said, 'And you trust him? He may not believe in the defect now, but once he's convinced—'

'He'll want exactly what *we* want. I'm sure of that.'

'OK. But are you sure IA won't be watching, too? If they've worked out why we're here, and they've bribed someone—'

Alison cut me off impatiently. 'There are still a few things you can't buy in this city. Spying on a military machine like Luminous would be suicidal. No one would risk it.'

'What about spying on unauthorised projects being run on a military machine? Maybe the crimes cancel out, and you end up a hero.'

She approached me, half naked, drying her face on my towel. 'We'd better hope not.'

I laughed suddenly. 'You know what I like most about Luminous? They're not really letting Exxon and McDonnell-Douglas use the same machine as the People's Liberation Army. Because the whole computer vanishes every time they pull the plug. There's no paradox at all, if you look at it that way.'

* * *

Alison insisted that we stand guard in shifts. Twenty-four hours earlier, I might have made a joke of it; now I reluctantly accepted the revolver she offered me, and sat watching the door in the neon-tinged darkness while she went out like a light.

The hotel had been quiet for most of the evening, but now it came to life. There were footsteps in the corridor every five minutes – and rats in the walls, foraging and screwing and probably giving birth. Police sirens wailed in the distance; a couple screamed at each other in the street below. I'd read somewhere that Shanghai was now the murder capital of the world – but was that *per capita*, or in absolute numbers?

After an hour, I was so jumpy that it was a miracle I hadn't blown my foot off. I unloaded the gun, then sat playing Russian roulette with the empty barrel. In spite of everything, I still wasn't ready to put a bullet in anyone's brain for the sake of defending the axioms of number theory.

* * *

Industrial Algebra had approached us in a perfectly civilised fashion. They were a small but aggressive UK-based company, designing specialised high-performance computing hardware for industrial and military applications. That they'd heard about the search was no great surprise – it had been openly discussed on the Internet for years, and even joked about in serious mathematical journals – but it seemed an odd coincidence when they made contact with us just days after Alison had sent me a private message from Zürich mentioning the latest 'promising' result. After half a dozen false alarms – all due to bugs and glitches – we'd stopped broadcasting the news of every unconfirmed find to the people who

were donating runtime to the project, let alone any wider circle. We were afraid that if we cried wolf one more time, half our collaborators would get so annoyed that they'd withdraw their support.

IA had offered us a generous slab of computing power on the company's private network – several orders of magnitude more than we received from any other donor. *Why?* The answer kept changing. Their deep respect for pure mathematics . . . their wide-eyed fun-loving attitude to life . . . their desire to be seen to be sponsoring a project so wild and hip and unlikely to succeed that it made SETI look like a staid blue-chip investment. It was, they'd finally 'conceded', a desperate bid to soften their corporate image after years of bad press for what certain unsavoury governments did with their really rather nice smart bombs.

We'd politely declined. They'd offered us highly paid consulting jobs. Bemused, we'd suspended all net-based calculations – and started encrypting our mail with a simple but highly effective algorithm Alison had picked up from Yuen.

Alison had been collating the results of the search on her own work station at her current home in Zürich, while I'd helped co-ordinate things from Sydney. No doubt IA had been eavesdropping on the incoming data, but they'd clearly started too late to gather the information needed to create their own map; each fragment of the calculations meant little in isolation. But when the work station was stolen (all the files were encrypted, it would have told them nothing) we'd finally been forced to ask ourselves: *If the defect turns out to be genuine, if the joke is no joke . . . then exactly what's at stake? How much money? How much power?*

On 7 June 2006, we met in a sweltering, crowded square in Hanoi. Alison wasted no time. She was carrying a backup of the data from the stolen work station in her notepad and she solemnly proclaimed that, this time, the defect was real.

The notepad's tiny processor would have taken centuries to repeat the long random trawling of the space of arithmetic statements which had been carried out on the net, but, led straight to the relevant computations, it could confirm the existence of the defect in a matter of minutes.

The process began with Statement S. Statement S was an assertion about some ludicrously huge numbers, but it wasn't mathematically sophisticated or contentious in any way. There were no claims here about infinite sets, no propositions concerning 'every integer'. It merely stated that a certain (elaborate) calculation performed on certain (very large) whole numbers led to a certain result – in essence, it was no different from something like '5+3 = 4x2'. It might have taken me ten years to check it with a pen and paper, but I could have carried out the task with nothing but primary-school mathematics and a great deal of patience. A statement like this could not be undecidable; it had to be either true or false.

The notepad decided it was true.

Then the notepad took statement S . . . and, in 423 simple, impeccably logical steps, used it to prove not-S.

I repeated the calculations on my own notepad, using a different software package. The result was exactly the same. I gazed at the screen, trying to concoct a plausible reason why two different machines running two different programs could have failed in identical ways. There'd certainly been cases in the past of a single misprinted algorithm in a computing textbook spawning a thousand dud programs. But the operations here were too simple, too basic.

Which left only two possibilities. Either conventional arithmetic was intrinsically flawed, and the whole Platonic ideal of the natural numbers was ultimately self-contradictory; or Alison was right, and an alternative arithmetic had come to hold sway in a 'computationally remote' region, billions of years ago.

I was badly shaken, but my first reaction was to try to play down the significance of the result. 'The numbers being manipulated here are greater than the volume of the observable universe, measured in cubic Planck lengths. If IA were hoping to use this on their foreign-exchange transactions, I think they've made a slight error of scale.' Even as I spoke, though, I knew it wasn't that simple. The raw numbers might have been trans-astronomical, but it was the mere 1,024 bits of the notepad's binary representations which had actually, physically misbehaved. Every truth in mathematics was

encoded, reflected, in countless other forms. If a paradox like this – which at first glance sounded like a dispute about numbers too large to apply even to the most grandiose cosmological discussions – could affect the behaviour of a five-gram silicon chip, then there could easily be a billion other systems on the planet at risk of being touched by the very same flaw.

But there was worse to come.

The theory was, we'd located part of the boundary between two incompatible systems of mathematics, both of which were *physically true*, in their respective domains. Any sequence of deductions which stayed entirely on one side of the defect – whether it was the 'near side', where conventional arithmetic applied, or the 'far side', where the alternative took over – would be free from contradictions. But any sequence which crossed the border would give rise to absurdities: hence S could lead to not-S.

So, by examining a large number of chains of inference, some of which turned out to be self-contradictory and some not, it should have been possible to map the area around the defect precisely – to assign every statement to one system or the other.

Alison displayed the first map she'd made. It portrayed an elaborately crenellated fractal border, rather like the boundary between two microscopic ice crystals – as if the two systems had been diffusing out at random from different starting points, and then collided, blocking each other's way. By now, I was almost prepared to believe that I really was staring at a snapshot of the creation of mathematics – a fossil of primordial attempts to define the difference between truth and falsehood.

Then she produced a second map of the same set of statements, and overlaid the two. The defect, the border, had shifted – advancing in some places, retreating in others.

My blood went cold. '*That* has got to be a bug in the software.'

'It's not.'

I inhaled deeply, looking around the square – as if the heedless crowd of tourists and hawkers, shoppers and executives, might offer some simple 'human' truth more resilient than mere arithmetic. But all I could think of was *1984*: Winston Smith, finally beaten into

submission, abandoning every touchstone of reason by conceding that *two and two make five*.

I said, 'OK. Go on.'

'In the early universe, some physical system must have tested out mathematics which was isolated, cut off from all the established results, leaving it free to decide the outcome at random. That's how the defect arose. But by now, *all* the mathematics in this region has been tested, all the gaps have been filled in. When a physical system tests a theorem on the near side, not only has it been tested a billion times before, but all the *logically adjacent* statements around it have been decided, and they imply the correct result in a single step.'

'You mean . . . peer pressure from the neighbours? No inconsistencies allowed, you have to conform? If $x-1 = y-1$, and $x+1 = y+1$, then x is left with no choice but to equal y because there's nothing "near by" to support the alternative?'

'Exactly. Truth is determined locally. And it's the same, deep into the far side. The alternative mathematics has dominated there, and every test takes place surrounded by established theorems which reinforce each other, and the "correct" – non-standard – result.'

'At the border, though—'

'At the border, every theorem you test is getting contradictory advice. From one neighbour, $x-1 = y-1$ . . . but, from another, $x+1 = y+2$. And the topology of the border is so complex that a near-side theorem can have more far-side neighbours than near-side ones – and vice versa.

'So the truth at the border isn't fixed, even now. Both regions can still advance or retreat. *It all depends on the order in which the theorems are tested.* If a solidly near-side theorem is tested first, and it lends support to a more vulnerable neighbour, that can guarantee that they both stay near-side.' She ran a brief animation which demonstrated the effect. 'But if the order is reversed, the weaker one *will* fall.'

I watched, light-headed. Obscure, but supposedly eternal, truths were tumbling like chess pieces. 'And . . . you think that physical processes going on *right now* – chance molecular events which keep inadvertently testing and re-testing different theories along the

border – cause each side to gain and lose territory?'

'Yes.'

'So there's been a kind of . . . random tide washing back and forth between the two kinds of mathematics, for the past few billion years?' I laughed uneasily, and did some rough calculations in my head. 'The expectation value for a random walk is the square root of N. I don't think we have anything to worry about. The tide isn't going to wash over any useful arithmetic in the lifetime of the universe.'

Alison smiled humourlessly, and held up the notepad again. 'The tide? No. But it's the easiest thing in the world to dig a channel. To bias the random flow.' She ran an animation of a sequence of tests which forced the far-side system to retreat across a small front – exploiting a 'beach-head' formed by chance, and then pushing on to undermine a succession of theorems. 'Industrial Algebra, though, I imagine, would be more interested in the reverse. Establishing a whole network of narrow channels of non-standard mathematics running deep into the realm of conventional arithmetic – which they could then deploy against theorems with practical consequences.'

I fell silent, trying to imagine tendrils of contradictory arithmetic reaching down into the everyday world. No doubt IA would aim for surgical precision, hoping to earn themselves a few billion dollars by corrupting the specific mathematics underlying certain financial transactions. But the ramifications would be impossible to predict – or control. There'd be no way to limit the effect, spatially. They could target certain mathematical truths, but they couldn't confine the change to any one location. *A few billion dollars, a few billion neurons, a few billion stars . . . a few billion people.* Once the basic rules of counting were undermined, the most solid and distinct objects could be rendered as uncertain as swirls of fog. This was not a power I would have entrusted to a cross between Mother Teresa and Carl Friedrich Gauss.

'So what do we do? Erase the map and just hope that IA never find the defect for themselves?'

'No.' Alison seemed remarkably calm – but then, her own long-cherished philosophy had just been confirmed, not razed to the

ground, and she'd had time on the flight from Zürich to think through all the *Realmathematik*. 'There's only one way to be sure that they can never use this. We have to strike first. We have to get hold of enough computing power to map the entire defect. And then we either iron the border flat, so it *can't* move: if you amputate all the pincers, there can be no pincer movements. Or – better yet, if we can get the resources – we push the border in, from all directions, and shrink the far-side system down to nothing.'

I hesitated. 'All we've mapped so far is a tiny fragment of the defect. We don't know how large the far side could be. Except that it can't be small, or the random fluctuations would have swallowed it long ago. And it *could* go on for ever; it could be infinite, for all we know.'

Alison gave me a strange look. 'You still don't get it, do you, Bruno? You're still thinking like a Platonist. The universe has only been around for fifteen billion years. It hasn't had time to create infinities. The far side *can't* go on for ever, because somewhere beyond the defect there are theorems which don't belong to *any* system. Theorems which have never been touched, never been tested, never been rendered true or false.

'And if we have to reach beyond the existing mathematics of the universe in order to surround the far side . . . then that's what we'll do. There's no reason why it shouldn't be possible, just so long as we get there first.'

* * *

When Alison took my place, at one in the morning, I was certain I wouldn't get any sleep. When she shook me awake three hours later, I still felt as though I hadn't.

I used my notepad to send a priming code to the data caches buried in our veins, and then we stood together side by side, left shoulder to right shoulder. The two chips recognised each other's magnetic and electrical signatures, interrogated each other to be sure, and then began radiating lower power microwaves. Alison's notepad picked up the transmission, and merged the two complementary data streams. The result was still heavily encrypted, but,

after all the precautions we'd taken so far, shifting the map into a hand-held computer felt about as secure as tattooing it onto our foreheads.

A taxi was waiting for us downstairs. The People's Institute for Advanced Optical Engineering was in Minhang, a sprawling technology park some thirty kilometres south of the city centre. We rode in silence through the grey pre-dawn light, past the giant ugly tower blocks thrown up by the landlords of the new millennium, riding out the fever as the necrotraps and their cargo dissolved into our blood.

As the taxi turned into an avenue lined with biotech and aerospace companies, Alison said, 'If anyone asks, we're PhD students of Yuen's, testing a conjecture in algebraic topology.'

'*Now* you tell me. I don't suppose you have any specific conjecture in mind? What if they ask us to elaborate?'

'On *algebraic topology*? At five o'clock in the morning?'

The Institute building was unimposing – sprawling black ceramic, three storeys high – but there was a five-metre electrified fence, and the entrance was guarded by two armed soldiers. We paid the taxi driver and approached on foot. Yuen had supplied us with visitor's passes, complete with photographs and fingerprints. The names were our own; there was no point indulging in unnecessary deception. If we were caught out, pseudonyms would only make things worse.

The soldiers checked the passes, then led us through an MRI scanner. I forced myself to breathe calmly as we waited for the results; in theory, the scanner could have picked up our symbionts' foreign proteins, lingering breakdown products from the necrotraps, and a dozen other suspicious trace chemicals. But it all came down to a question of what they were looking for; magnetic resonance spectra for billions of molecules had been catalogued, but no machine could hunt for all of them at once.

One of the soldiers took me aside and asked me to remove my jacket. I fought down a wave of panic and then struggled not to overcompensate: if I'd had nothing to hide, I would still have been nervous. He prodded the bandage on my upper arm; the surrounding skin was still red and inflamed. 'What's this?'

'I had a cyst there. My doctor cut it out, this morning.'

He eyed me suspiciously, and peeled back the adhesive bandage with ungloved hands. I couldn't bring myself to look; the repair cream should have sealed the wound completely – at worst there should have been old, dried blood – but I could *feel* a faint liquid warmth along the line of the incision.

The soldier laughed at my gritted teeth, and waved me away with an expression of distaste. I had no idea what he thought I might have been hiding, but I saw fresh red droplets beading the skin before I closed the bandage.

Yuen Ting-fu was waiting for us in the lobby. He was a slender, fit-looking man in his late sixties, casually dressed in denim. I let Alison do all the talking: apologising for our lack of punctuality (although we weren't actually late), and thanking him effusively for granting us this precious opportunity to pursue our unworthy research. I stood back and tried to appear suitably deferential. Four soldiers looked on impassively; they didn't seem to find all this grovelling excessive. And no doubt I would have been giddy with awe if I really had been a student granted time here for some run-of-the-mill thesis.

We followed Yuen as he strode briskly through a second checkpoint and scanner (this time, no one stopped us), then down a long corridor with a soft grey vinyl floor. We passed a couple of white-coated technicians, but they barely gave us a second glance. I'd had visions of a pair of obvious foreigners attracting as much attention here as we would have done wandering through a military base, but that was absurd. Half the runtime on Luminous was sold to foreign corporations – and because the machine was most definitely *not* linked to any communciations network, commercial users had to come here in person. Just how often Yuen wangled free time for his students – whatever their nationality – was another question, but if he believed it was the best cover for us, I was in no position to argue. I only hoped he'd planted a seamless trail of reassuring lies in the University records and beyond, in case the Institute administration decided to check up on us in any detail.

We stopped in at the operations room, and Yuen chatted with the technicians. Banks of flatscreens covered one wall, displaying status

histograms and engineering schematics. It looked like the control centre for a small particle accelerator – which wasn't far from the truth.

Luminous was, literally, a computer made of light. It came into existence when a vacuum chamber, a cube five metres wide, was filled with an elaborate standing wave created by three vast arrays of high-powered lasers. A coherent electron beam was fed into the chamber – and just as a finely machined grating built of solid matter could diffract a beam of light, a sufficiently ordered (and sufficiently intense) configuration of light could diffract a beam of matter.

The electrons were redirected from layer to layer of the light cube, recombining and interfering at each stage, every change in their phase and intensity performing an appropriate computation – and the whole system could be reconfigured, nanosecond by nanosecond, into complex new 'hardware' optimised for the calculations at hand. The auxilliary supercomputers controlling the laser arrays could design, and then instantly build, the perfect machine of light to carry out each particular stage of any program.

It was, of course, fiendishly difficult technology, incredibly expensive and temperamental. The chance of ever putting it on the desktops of Tetris-playing accountants was zero, so nobody in the West had bothered to pursue it.

And this cumbersome, unwieldy, impractical machine ran faster than all the pieces of silicon hanging off the Internet, combined.

We continued on to the programming room. At first glance, it might have been the computing centre in a small primary school, with half a dozen perfectly ordinary work stations sitting on white Formica tables. They just happened to be the only six in the world which were hooked up to Luminous.

We were alone with Yuen now. Alison cut the protocol and just glanced briefly in his direction for approval, before hurriedly linking her notepad to one of the work stations and uploading the encrypted map. As she typed in the instructions to decode the file, all the images running through my head of what would have happened if I'd poisoned the soldier at the gate receded into insignificance. We now had half an hour to banish the defect, and we still had no idea how far it extended.

Yuen turned to me; the tension on his face betrayed his own anxieties, but he mused philosophically, 'If our arithmetic seems to fail for these large numbers, does it mean the mathematics, the ideal, is really flawed and mutable, or only that the behaviour of matter always falls short of the ideal?'

I replied, 'If every class of physical objects "falls short" in exactly the same way, whether it's boulders or electrons or abacus beads, what is it that their common behaviour is obeying – or defining – if not the mathematics?'

He smiled, puzzled. 'Alison seemed to think you were a Platonist.'

'Lapsed. Or . . . defeated. I don't see what it can *mean* to talk about standard number theory still being true for these statements – in some vague Platonic sense – if no real objects can ever reflect that truth.'

'We can still imagine it. We can still contemplate the abstraction. It's only the physical act of validation that must fall through. Think of transfinite arithmetic: no one can physically test the properties of Cantor's infinities, can they? We can only reason about them from afar.'

I didn't reply. Since the revelations in Hanoi, I'd pretty much lost faith in my power to 'reason from afar' about anything I couldn't personally describe with Arabic numerals on a single sheet of paper. Maybe Alison's idea of 'local truth' was the most we could hope for; anything more ambitious was beginning to seem like the comic-book 'physics' of swinging a rigid beam ten billion kilometres long around your head, and predicting that the far end would exceed the speed of light.

An image blossomed on the work-station screen: it began as the familiar map of the defect, but Luminous was already extending it at a mind-boggling rate. Billions of inferential loops were being spun around the margins: some confirming their own premises, and thus delineating regions where a single, consistent mathematics held sway; others skewing into self-contradiction, betraying a border crossing. I tried to imagine what it would have been like to follow one of those Möbius-strips of deductive logic in my head; there were no difficult concepts involved, it was only the sheer size of the statements which made that impossible. But would the contradictions have driven me

into gibbering insanity, or would I have found every step perfectly reasonable, and the conclusion simply unavoidable? Would I have ended up calmly, happily conceding: *Two and two make five?*

As the map grew – smoothly re-scaled to keep it fitting on the screen, giving the unsettling impression that we were retreating from the alien mathematics as fast as we could, and only just avoiding being swallowed – Alison sat hunched forward, waiting for the big picture to be revealed. The map portrayed the network of statements as an intricate lattice in three dimensions (a crude representational convention, but it was as good as any other). So far, the border between the regions showed no sign of overall curvature, just variously sized random incursions in both directions. For all we knew, it was possible that the far-side mathematics enclosed the near side completely, that the arithmetic we'd once believed stretched out to infinity was really no more than a tiny island in an ocean of contradictory truths.

I glanced at Yuen; he was watching the screen with undisguised pain. He said, 'I read your software, and I thought: Sure, this looks fine, but some glitch on your machines is the real explanation. Luminous will soon put you right.'

Alison broke in jubilantly, 'Look, it's turning!'

She was right. As the scale continued to shrink, the random fractal meanderings of the border were finally being subsumed by an overall convexity – a convexity of the far side. It was as if the viewpoint was backing away from a giant spiked sea-urchin. Within minutes, the map showed a crude hemisphere, decorated with elaborate crystalline extrusions at every scale. The sense of observing some palaeomathematical remnant was stronger than ever now: this bizarre cluster of theorems really did look as if it had exploded out from some central premise into the vacuum of unclaimed truths, perhaps a billionth of a second after the Big Bang, only to be checked by an encounter with our own mathematics.

The hemisphere slowly extended into a three-quarters sphere . . . and then a spiked whole. The far side was bounded, finite. It was the island, not us.

Alison laughed uneasily. 'Was that true before we started, or did

we just make it true?' *Had the near side enclosed the far side for billions of years, or had Luminous broken new ground, actively extending the near side into mathematical territory which had never been tested by any physical system before?*

We'd never know. We'd designed the software to advance the mapping along a front in such a way that any unclaimed statements would be instantly recruited into the near side. If we'd reached out blindly, far into the void, we might have tested an isolated statement – and inadvertently spawned a whole new alternative mathematics to deal with.

Alison said, 'OK, now we have to decide. Do we try to seal the border or do we take on the whole structure?'

The software, I knew, was busy assessing the relative difficulty of the tasks.

Yuen replied at once, 'Seal the border, nothing more. You mustn't destroy this.' He turned to me, imploringly. 'Would you smash up a fossil of *Australopithecus*? Would you wipe the cosmic background radiation out of the sky? This may shake the foundations of all my beliefs, but it encodes the truth about our history. We have no right to obliterate it, like vandals.'

Alison eyed me nervously. *What was this, majority rule?* Yuen was the only one with any power here; he could pull the plug in an instant. And yet it was clear from his demeanour that he wanted a consensus; he wanted our moral support for any decision.

I said cautiously, 'If we smooth the border, that'll make it literally impossible for IA to exploit the defect, won't it?'

Alison shook her head. 'We don't know that. There may be a quantum-like component of spontaneous defections, even for statements which appear to be in perfect equilibrium.'

Yuen countered, 'Then there could be spontaneous defections *anywhere*, even far from any border. Erasing the whole structure will guarantee nothing.'

'It will guarantee that IA won't find it! Maybe pinpoint defections *do* occur, all the time, but the next time they're tested they'll always revert. They're surrounded by explicit contradictions; they have no chance of getting a foothold. You can't compare a few transient

glitches with this . . . *armoury* of counter-mathematics!'

The defect bristled on the screen like a giant caltrap. Alison and Yuen both turned to me expectantly. As I opened my mouth, the work station chimed. The software had examined the alternatives in detail: destroying the entire far side would take Luminous twenty-three minutes and seventeen seconds – about a minute less than the time we had left. Sealing the border would take more than an hour.

I said, 'That can't be right.'

Alison groaned. 'But it is! There's random interference going on at the border from other systems all the time – and doing anything finicky there means coping with that noise, fighting it. Charging ahead and pushing the border inwards is different: you can exploit the noise to speed the advance. It's not a question of *dealing with a mere surface* versus *dealing with a whole volume*. It's more like . . . trying to carve an island into an absolutely perfect circle while waves are constantly crashing on the beach – versus bulldozing the whole thing into the ocean.'

We had thirty seconds to decide, or we'd be doing neither today. And maybe Yuen had the resources to keep the map safe from IA while we waited a month or more for another session on Luminous, but I wasn't prepared to live with that kind of uncertainty.

'I say we get rid of the whole thing. Anything less is too dangerous. Future mathematicians will still be able to study the map – and if no one believes that the defect itself ever really existed, that's just too bad. IA are too close. We can't risk it.'

Alison had one hand poised above the keyboard. I turned to Yuen; he was staring at the floor with an anguished expression. He'd let us state our views, but in the end it was his decision.

He looked up, and spoke sadly but decisively.

'OK. Do it.'

Alison hit the key, with about three seconds to spare. I sagged into my chair, light-headed with relief.

* * *

We watched the far side shrinking. The process didn't look quite as crass as *bulldozing an island* – more like dissolving some quirkily

beautiful crystal in acid. Now that the danger was receding before our eyes, though, I was beginning to suffer faint pangs of regret. Our mathematics had coexisted with this strange anomaly for fifteen billion years, and it shamed me to think that, within months of its discovery, we'd backed ourselves into a corner where we'd had no choice but to destroy it.

Yuen seemed transfixed by the process. 'So are we breaking the laws of physics – or enforcing them?'

Alison said, 'Neither. We're merely changing what the laws imply.'

He laughed softly. '"Merely". For some esoteric set of complex systems, we're rewriting the high-level rules of their behaviour. Not including the human brain, I hope.'

My skin crawled. 'Don't you think that's . . . unlikely?'

'I was joking.' He hesitated, then added soberly, 'Unlikely for humans, but *someone* could be relying on this, somewhere. We might be destroying the whole basis of their existence: certainties as fundamental to them as a child's multiplication tables are to us.'

Alison could barely conceal her scorn. 'This is junk mathematics – a relic of a pointless accident. Any kind of life which evolved from simple to complex forms would have no use for it. Our mathematics works for . . . rocks, seeds, animals in the herd, members of the tribe. *This* only kicks in beyond the number of particles in the universe—'

'Or smaller systems which represent those numbers,' I reminded her.

'And you think life somewhere might have a burning need to do *non-standard trans-astronomical arithmetic*, in order to survive? I doubt that very much.'

We fell silent. Guilt and relief could fight it out later, but no one suggested halting the program. In the end, maybe nothing could outweigh the havoc the defect would have caused if it had ever been harnessed as a weapon, and I was looking forward to composing a long message to Industrial Algebra, informing them of precisely what we'd done to the object of their ambitions.

Alison pointed to a corner of the screen. 'What's that?' A narrow dark spike protruded from the shrinking cluster of statements. For a

moment I thought it was merely avoiding the near side's assault, but it wasn't. It was slowly, steadily, growing longer.

'Could be a bug in the mapping algorithm.' I reached for the keyboard and zoomed in on the structure. In close-up, it was several thousand statements wide. At its border, Alison's program could be seen in action, testing statements in an order designed to force tendrils of the near side ever deeper into the interior. This slender extrusion, ringed by contradictory mathematics, should have been corroded out of existence in a fraction of a second. Something was actively countering the assault, though – repairing every trace of damage before it could spread.

'If IA have a bug here—' I turned to Yuen. 'They couldn't take on Luminous directly, so they couldn't stop the whole far side shrinking, but a tiny structure like this . . . What do you think? Could they stabilise it?'

'Perhaps,' he conceded. 'Four or five hundred top-speed work stations could do it.'

Alison was typing frantically on her notepad. She said, 'I'm writing a patch to identify any systematic interference and divert all our resources against it.' She brushed her hair out of her eyes. 'Look over my shoulder, will you, Bruno? Check me as I go.'

'OK.' I read through what she'd written so far. 'You're doing fine. Stay calm.' Her hands were trembling.

The spike continued to grow steadily. By the time the patch was ready, the map was re-scaling constantly to fit it on the screen.

Alison triggered the patch. An overlay of electric blue appeared along the spike, flagging the concentration of computing power, and the spike abruptly froze.

I held my breath, waiting for IA to notice what we'd done – and switch their resources elsewhere? If they did, no second spike would appear – they'd never get that far – but the blue marker on the screen would shift to the site where they'd regrouped and tried to make it happen.

But the blue glow didn't move from the existing spike. And the spike didn't vanish under the weight of Luminous's undivided efforts.

Instead, it began to grow again, slowly.

Yuen looked ill. 'This is *not* Industrial Algebra. There's no computer on the planet—'

Alison laughed derisively. 'What are you saying now? Aliens who need the far side are defending it? Aliens *where*? Nothing we've done has had time to reach even . . . Jupiter.' There was an edge of hysteria in her voice.

'Have you measured how fast the changes propagate? Do you know, for certain, that they can't travel faster than light – with the far-side mathematics undermining the logic of relativity?'

I said, 'Whoever it is, they're not defending all their borders. They're putting everything they've got into the spike.'

'They're aiming at something. A specific target.' Yuen reached over Alison's shoulder for the keyboard. 'We're shutting this down. Right now.'

She turned on him, blocking his way. 'Are you crazy? We're almost holding them off! I'll rewrite the program, fine-tune it, get an edge in efficiency—'

'No! We stop threatening them, then see how they react. We don't know what harm we're doing—'

He reached for the keyboard again.

Alison jabbed him in the throat with her elbow, hard. He staggered backwards, gasping for breath, then crashed to the floor, bringing a chair down on top of him. She hissed at me, 'Quick – shut him up!'

I hesitated, loyalties fracturing; his idea had sounded perfectly sane to me. But if he started yelling for security . . .

I crouched down over him, pushed the chair aside, then clasped my hand over his mouth, forcing his head back with pressure on the lower jaw. We'd have to tie him up and then try brazenly marching out of the building without him. But he'd be found in a matter of minutes. Even if we made it past the gate, we were screwed.

Yuen caught his breath and started struggling; I clumsily pinned his arms with my knees. I could hear Alison typing, a ragged staccato; I tried to get a glimpse of the work-station screen, but I couldn't turn that far without taking my weight off Yuen.

I said, 'Maybe he's right: maybe we should pull back and see what happens.' *If the alterations could propagate faster than light . . . how many distant civilisations might have felt the effects of what we'd done?* Our first contact with extraterrestrial life could turn out to be an attempt to obliterate mathematics which they viewed as . . . what? A precious resource? A sacred relic? An essential component of their entire world view?

The sound of typing stopped abruptly. 'Bruno? Do you feel—'

'What?'

Silence.

'What?'

Yuen seemed to have given up the fight. I risked turning around.

Alison was hunched forward, her face in her hands. On the screen, the spike had ceased its relentless linear growth, but now an elaborate dendritic structure had blossomed at its tip. I glanced down at Yuen; he seemed dazed, oblivious to my presence. I took my hand from his mouth, warily. He lay there placidly, smiling faintly, eyes scanning something I couldn't see.

I climbed to my feet. I took Alison by the shoulders and shook her gently; her only response was to press her face harder into her hands. The spike's strange flower was still growing, but it wasn't spreading out into new territory; it was sending narrow shoots back in on itself, criss-crossing the same region again and again with ever finer structures.

Weaving a net? Searching for something?

It hit me with a jolt of clarity more intense than anything I'd felt since childhood. It was like reliving the moment when the whole concept of *numbers* had finally snapped into place – but with an adult's understanding of everything it opened up, everything it implied. It was a lightning-bolt revelation – but there was no taint of mystical confusion: no opiate haze of euphoria, no pseudo-sexual rush. In the clean-lined logic of the simplest concepts, I saw and understood exactly how the world worked—

—Except that it was all wrong, it was all false, it was all impossible.

Quicksand.

Assailed by vertigo, I swept my gaze around the room, counting

frantically: *Six work stations. Two people. Six chairs.* I grouped the work stations: three sets of two, two sets of three. One and five, two and four; four and two, five and one.

I weaved a dozen cross-checks for consistency – *for sanity* . . . but everything added up.

They hadn't stolen the old arithmetic; they'd merely blasted the new one into my head, on top of it.

Whoever had resisted our assault with Luminous had reached down with the spike and rewritten our neural metamathematics – the arithmetic which underlay our own reasoning *about* arithmetic – enough to let us glimpse what we'd been trying to destroy.

Alison was still uncommunicative, but she was breathing slowly and steadily. Yuen seemed fine, lost in a happy reverie. I relaxed slightly, and began trying to make sense of the flood of far-side arithmetic surging through my brain.

On their own terms, the axioms were . . . trivial, obvious. I could see that they corresponded to elaborate statements about trans-astronomical integers, but performing an exact translation was far beyond me; and thinking about the entities they described in terms of the huge integers they represented was a bit like thinking about *pi* or *the square root of two* in terms of the first ten thousand digits of their decimal expansion: it would be missing the point entirely. These alien 'numbers' – the basic objects of the alternative arithmetic – had found a way to embed themselves in the integers, and to relate to each other in a simple, elegant way, and if the messy corollaries they implied upon translation contradicted the rules integers were supposed to obey . . . well, only a small, remote patch of obscure truths had been subverted.

Someone touched me on the shoulder. I started, but Yuen was beaming amiably, all arguments and violence forgotten.

He said, 'Lightspeed is *not* violated. All the logic which requires that remains intact.' I could only take him at his word; the result would have taken me hours to prove. Maybe the aliens had done a better job on him, or maybe he was just a superior mathematician in either system.

'Then . . . where are they?' At lightspeed, our attack on the far side

could not have been felt any further away than Mars, and the strategy used to block the corrosion of the spike would have been impossible with even a few seconds' time lag.

'The atmosphere?'

'You mean *Earth's*?'

'Where else? Or maybe the oceans.'

I sat down heavily. Maybe it was no stranger than any conceivable alternative, but I still baulked at the implications.

Yuen said, 'To us, their structure wouldn't look like "structure" at all. The simplest unit might involve a group of thousands of atoms – representing a trans-astronomical number – not necessarily even *bonded together* in any conventional way, but breaking the normal consequences of the laws of physics, obeying a different set of high-level rules which arise from the alternative mathematics. People have often mused about the chances of intelligence being coded into long-lived vortices on distant gas giants . . . but *these* creatures won't be in hurricanes or tornadoes. They'll be drifting in the most innocuous puffs of air, invisible as neutrinos.'

'Unstable—'

'Only according to our mathematics. Which does not apply.'

Alison broke in suddenly, angrily. 'Even if all of this is true, where does it get us? Whether the defect supports a whole invisible ecosystem or not, IA will still find it, and use it, in exactly the same way.'

For a moment I was dumbstruck. *We were facing the prospect of sharing the planet with an undiscovered civilisation and all she could think about was IA's grubby machinations?*

She was absolutely right, though. Long before any of these extravagant fantasies could be proved or disproved, IA could still do untold harm.

I said, 'Leave the mapping software running, but shut down the shrinker.'

She glanced at the screen. 'No need. They've overpowered it, or undermined its mathematics.' The far side was back to its original size.

'Then there's nothing to lose. Shut it down.'

She did. No longer under attack, the spike began to reverse its growth. I felt a pang of loss as my limited grasp of the far-side mathematics suddenly evaporated; I tried to hold on, but it was like clutching at air.

When the spike had retracted completely, I said, 'Now we try doing an Industrial Algebra. We try bringing the defect closer.'

We were almost out of time, but the task was easy enough. In thirty seconds, we rewrote the shrinking algorithm to function in reverse.

Alison programmed a function key with the commands to revert to the original version, so that if the experiment backfired, one keystroke would throw the full weight of Luminous behind a defence of the near side again.

Yuen and I exchanged nervous glances. I said, 'Maybe this wasn't such a good idea.'

Alison disagreed. 'We need to know how they'll react to this. Better we find out now than leave it to IA.'

She started the program running.

The sea-urchin began to swell, slowly. I broke out in a sweat. The far-siders hadn't harmed us, so far, but this felt like tugging hard at a door which you really, badly, didn't want to see thrown open.

A technician poked her head into the room and announced cheerfully, 'Down for maintenance in two minutes!'

Yuen said, 'I'm sorry, there's nothing—'

The whole far side turned electric blue. Alison's original patch had detected a systematic intervention.

We zoomed in. Luminous was picking off vulnerable statements of the near side, but something else was repairing the damage.

I let out a strangled noise that might have been a cheer.

Alison smiled serenely. She said, 'I'm satisfied. IA don't stand a chance.'

Yuen mused, 'Maybe they have a reason to defend the status quo. Maybe they rely on the border itself, as much as the far side.'

Alison shut down our reversed shrinker. The blue glow

vanished; both sides were leaving the defect alone. There were a thousand questions we all wanted answered, but the technicians had thrown the master switch, and Luminous itself had ceased to exist.

* * *

The sun was breaking through the skyline as we rode back into the city. As we pulled up outside the hotel, Alison started shaking and sobbing. I sat beside her, squeezing her hand. I knew she'd felt the weight of what might have happened, all along, far more than I had.

I paid the driver, and then we stood on the street for a while, silently watching the cyclists go by, trying to imagine how the world would change as it endeavoured to embrace this new contradiction between the exotic and the mundane, the pragmatic and the Platonic, the visible and the invisible.

Mister Volition

'Give me the patch.'

He hesitates, despite the gun, long enough to confirm that the thing must be genuine. He's cheaply dressed but expensively groomed: manicured and depilated, with the baby-smooth skin of rich middle age. Any card in his wallet would be p-cash only, anonymous but encrypted, useless without his own living fingerprints. He's wearing no jewellery, and his watchphone is plastic; the patch is the only thing worth taking. Good fakes cost 15 cents, good real ones 15 K, but he's the wrong age, and the wrong class, to want to wear a fake for the sake of fashion.

He tugs at the patch gently, and it dislodges itself from his skin; the adhesive rim doesn't leave the faintest weal or pluck a single hair from his eyebrow. His newly naked eye doesn't blink or squint, but I know it's not truly sighted yet; the suppressed perceptual pathways take hours to reawaken.

He hands me the patch; I half expect it to stick to my palm, but it doesn't. The outer face is black, like anodised metal, with a silver-grey logo of a dragon in one corner – drawn 'escaping' from a cut-and-folded drawing of itself, to bite its own tail. Recursive Visions, after Escher. I press the gun harder against his stomach to remind him of its presence, while I glance down and turn the thing over. The inner face appears velvet black at first, but as I tilt it I catch the reflection of a street light, rainbow-diffracted by the array of quantum-dot lasers. Some plastic fakes are moulded with pits which give a similar effect, but the sharpness of this image – dissected into colours, but not blurred at all – is like nothing I've ever seen before.