

## THE MU PUZZLE

MATH 105-6: FIRST-YEAR SEMINAR: THEORIES OF MIND AND MATHEMATICS  
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The following exercises are based on Chapter 1 of:

- Douglas R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, 1979.

### BASIC VOCABULARY AND GRAMMAR OF MIU

The purpose of this exercise is to study an artificial language called “Miu.” It has three words:

**Definition (word):** A *word* in Miu is any of the following three symbols (when writing in Miu, we will always use a non-serif font):

M, I, U

You should think of these as logographs (e.g. Japanese kanji or Chinese characters) rather than symbols from an alphabet (e.g. Cyrillic) or syllabary (e.g. Japanese kana). An English-to-Miu dictionary would have only three entries. I won’t provide an English-to-Miu dictionary (indeed, I don’t own one), but some of the meanings of Miu will become apparent.

**Definition (string):** A *string* (also called a *formula*) is any list of Miu words.

**Examples:** The following are all strings:

M, IUI, UU, MMMUM

On the other hand, XMA is not a string in Miu, since neither X nor A is a word in Miu.

We can now define the *grammar* of our artificial language.

**Definition (grammatical sentence):** A string is a *grammatical sentence* (also called a *well-formed formula*) if its first word is M, and no other words are Ms.

**Examples:** The following strings are grammatical sentences:

M, MIUIU, MUUIII

The following strings are not grammatical:

U, UIUMM, MUM

Thus the *meaning* of the word M is purely *syntactic*: it means “this is the start of a sentence (and the end of any previous sentence).” **Note:** *Syntax* is essentially the same as *grammar*. For comparison, *semantics* means “meaning” — the things “in the real world” that words signify. The words I and U are purely *semantic*.

We will call any list of grammatical sentences a *paragraph*.

**Exercise 1:** Which strings are paragraphs? Is there unambiguous *parsing* (i.e. breaking of a paragraph into sentences)?

## SOUND ARGUMENTS

In any language, not all grammatical sentences express truths. To convince someone of the truth of a sentence, you should give a *sound argument*. Recall that an argument is *valid* if every step follows logically from the previous steps. But that's not good enough: you could start with a false premise. An argument is *sound* if it is both valid and also every premise is true.

In Miu, there is one sentence that everyone knows to be true (agreed-upon sentences are called *axioms*):

**Axiom:** MI

Since we don't know what I means, we don't know what this sentence means — it could mean, for example, “There is no God but Allah,” or maybe it means “ $0 = 0$ .”

There are four *rules of inference*, which are the allowed steps in a valid argument.

**Rule 1:** If  $x$  is any string, then it is valid to proceed from  $MxI$  to  $MxIU$ .

**Note:** The italic letter “ $x$ ” is not a string in Miu. What the rule means is that if you choose any string in Miu, and replace all instances of  $x$  with that string, then you get an instance of the rule.

**Example:** It is valid to go from the claim MUUI to the conclusion MUUIU.

**Rule 2:** If  $x$  is any string, then from  $Mx$  it is valid to conclude  $Mxx$ .

**Example:** MUMUU is a valid paragraph. I.e. you may validly follow the sentence MU by the sentence MUU.

**Rule 3:** If III occurs anywhere in a sentence, it is valid to replace it by U.

**Example:** MUIIIUMUIUU and MUIIIUMUUIU are both valid paragraphs.

**Rule 4:** If UU occurs anywhere in a sentence, it is valid to delete it.

**Example:** MUUUUMUUM is a valid paragraph consisting of three sentences.

All together, a paragraph is *sound* if it is valid and also its first sentence is MI.

**Exercise 2:** Come up with some sound arguments to get a feel for the rules of inference.

## JUSTIFIABLE KNOWLEDGE

**Definition (justifiable):** A sentence is *justifiable* if there is some sound argument (the *justification*) with that sentence as its conclusion. (Justifiable sentences are also called *theorems*; their justifications are also called *proofs*.)

**Example:** MIUIIU is justifiable, because the paragraph MIMIIMIUMIUIIU is sound.

**Main exercise:** Is MU justifiable?