Deformation Quantization
Exercise Sheet 4

Ján Pulmann

$^!$ *marks a more important exercise*

Exercise 1.

1. Check that L_∞ morphisms take Maurer-Cartan elements to Maurer-Cartan elements
2. $^!$ Check that an L_∞ morphism between 2 dglas induces a morphism of Lie algebras:
$$U_1 : H^*(\mathfrak{g}_1, d_1) \longrightarrow H^*(\mathfrak{g}_2, d_2)$$
3. *(optional):* Check that U_1 defined by the graph expansions agree with the HKR isomorphism (use that the integral corresponding to the graph with one aerial and m ground vertices is $(2\pi)^m/m!$). If you wish, you can compute this integral for low m.

Exercise 2. Let g be a Lie algebra and let $t \in (\mathrm{Sym}^2\mathfrak{g})^\mathfrak{g}$. Let (X, ρ_X) and (Y, ρ_Y) be representations of $\mathfrak{g}$.

- $^!$ Check that $t_{X,Y} : X \otimes Y \longrightarrow X \otimes Y$ defined as below assemble into a natural transformation (endomorphism of the tensor product functor of $U\mathfrak{g}$-mod)
$$t_{X,Y} : x \otimes y \longmapsto t^{ij} \rho_X(e_i)(x) \otimes \rho_Y(e_j)(y).$$
Here $t = t^{ij} e_i \otimes e_j$ in some basis $\{e_i\}$ of $\mathfrak{g}$. Where do we use the invariance of t?
- Check that
$$\beta_{X,Y} = \sigma_{X,Y} \circ (id_{X\otimes Y} + \varepsilon t_{X,Y}) : X \otimes Y \to Y \otimes X$$
gives a braiding on $U\mathfrak{g}\text{-mod}_\varepsilon$, where $\varepsilon^2 = 0$ is freely adjoined to Hom-spaces of $U\mathfrak{g}$-mod. The natural transformation σ is the symmetry of $U\mathfrak{g}$-mod.