Introduction to algebraic K-theory
Problem set 2

Ex. 1. Let k be a field, and $n \in \mathbb{N}$. Consider $R = k[x]/\langle x^n \rangle$.

1. Prove that R is a local ring, that is, that it has a unique maximal ideal.
2. Compute (actually compute) $K_1(R)$.

Ex. 2. Pick two of the equations defining a simplicial set, and draw the pictorial representation for a choice of n, i, j, as we did in lecture—for your own sanity, you should keep $n \leq 3$.

Ex. 3. This exercise will justify why you can think of the category of categories Cat "inside" the category of simplicial sets sSet.

1. Think about how the nerve functor $N\colon \mathrm{Cat} \to \mathrm{sSet}$ induces a function

$$\mathrm{Hom}(\mathcal{C}, \mathcal{C}') \to \mathrm{Hom}(N\mathcal{C}, N\mathcal{C}')$$

2. Prove that this function is injective.
3. Prove, or sketch the proof, that it is also surjective.