

Global Categorical Symmetries & ad for Perimeter Scholars International

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Undergraduate Pizza Seminar, U. of Alberta, 5 Nov 2025

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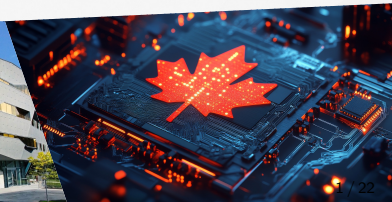
<https://categorified.net/UG-Alberta.pdf>



About Perimeter Institute for Theoretical Physics

Our Vision

To create the world's foremost centre for research, graduate training, and educational outreach in theoretical physics, uniting public and private partners, and the world's best scientific minds, in a shared enterprise to achieve breakthroughs that will transform our future.



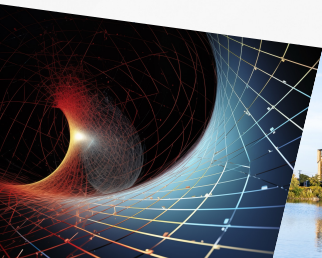
About Perimeter Institute for Theoretical Physics

Our Mission

Perimeter Institute for Theoretical Physics is an **independent, resident-based research institute** devoted to foundational issues in theoretical physics at the highest levels of **international excellence**.

We strive to create a lively and **dynamic research atmosphere** where many approaches to fundamental questions, both **orthodox and unorthodox**, are pursued simultaneously, and where a balance between formal and phenomenological research is established.

We are committed to collaborating with the surrounding academic community by providing **outstanding educational and research opportunities** for graduate students. Our **world-class outreach program** conveys the wonder and mystery of the universe and the importance of future scientific breakthroughs to the public in Canada and beyond.



Perimeter by the numbers

27 Full-time research faculty

20 Associate faculty (cross-appointed with partner universities)

3 Teaching faculty

1 Outreach faculty

10 International research & training partnerships

21 Canadian research & training partnerships

101 Canadian affiliates



58 Postdoctoral researchers

63 PhD students

25 Masters students through Perimeter Scholars International (PSI)

15 Conferences & workshops per year

1,800 International visitors per year

About Perimeter Institute for Theoretical Physics

Perimeter's Core Activities

RESEARCH

Cutting-edge theoretical physics for tomorrow's great breakthroughs.

TRAINING

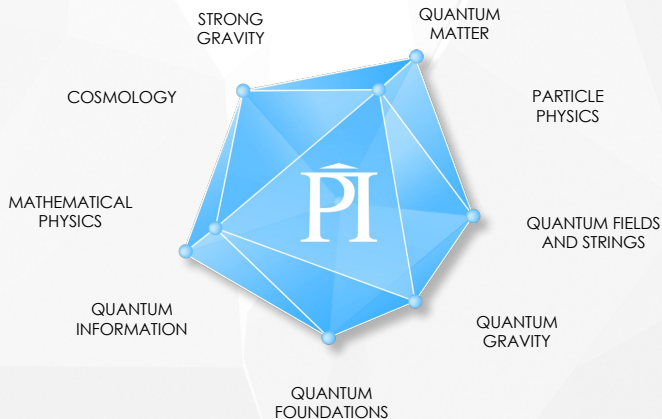
The next generation of scientists, leaders, and innovators.

OUTREACH

Activities that foster a love of science in our schools and communities.



Perimeter Research Areas



About Perimeter Institute for Theoretical Physics

Outreach Activities

TEACHING TEACHERS

52,000+

Participants in PI's Teacher
Network workshops

PUBLIC LECTURES

155+

talks by leading experts
teaching everyday people
about physics

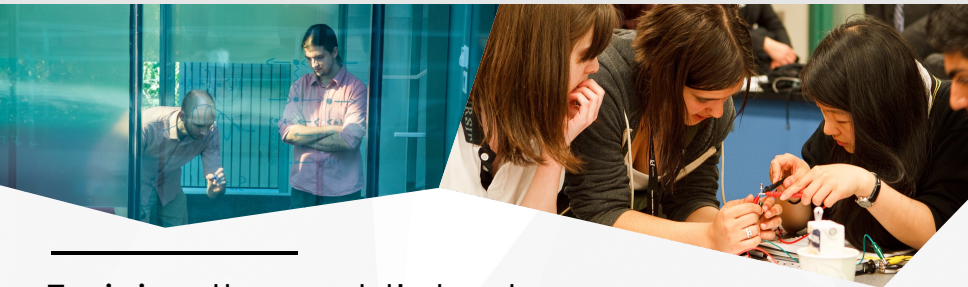
VIDEOS & PODCASTS

500,000+ podcast
views/listens on all channels

Millions views on YouTube



About Perimeter Institute for Theoretical Physics



Training the world's best

Perimeter attracts and develops the next generation of brilliant minds. From our world-leading PhD and postdoctoral programs to our undergraduate summer school and master's program, we are training tomorrow's leaders and innovators.

1,300+

Students & postdocs
trained since 2006

420

PSI students
in 15 years

900+

PhD students
from over 29 countries
since inception

Apply by February 1!

Perimeter Scholars International



UNIVERSITY OF
WATERLOO



PERIMETER
SCHOLARS
INTERNATIONAL

- Perimeter's master's graduate program
- One-year program with an optional second-year co-op component
- Producing critical thinkers with widely applicable skills in emerging fields



About Perimeter Scholars International

- ▶ **Core courses:** foundational graduate-level subjects including classical physics, quantum theory, general relativity, quantum field theory, statistical physics, mathematical physics, and numerical methods.
- ▶ **Elective courses:** modern topics from fields such as cosmology, particle physics, gravitational physics, string theory, quantum information, quantum matter, and cutting-edge research topics from various specialized fields.
- ▶ **Winter school:** a week-long, hands-on group research project in collaboration with selected researchers.
- ▶ **Essay:** a research project supervised by a faculty member, producing an essay that is publicly presented and defended at the end of the PSI program.

About Perimeter Scholars International

- ▶ **Assessment:** conducted by the academic staff as oral interviews (for core courses), homework assignments, and tutorial participation. All course grades are either “credit” or “no credit.” The goal is to encourage all students to achieve their potential and to avoid grade-chasing competition.
- ▶ **Degrees:** Master of Science in Physics degree from the University of Waterloo, as well as a Perimeter Scholars International certificate from Perimeter Institute for Theoretical Physics. The PSI certificate is given to students at the end of the program, while the master’s degree is awarded at the university’s convocation ceremony following the end of the academic term.

Apply by February 1!

About Perimeter's home in Waterloo

Quantum Valley: Waterloo

Perimeter sits at the heart of Waterloo's Quantum Valley, a unique innovation ecosystem where theoretical research leads to experiment, experiment leads to technology, and technology leads to commercialization.

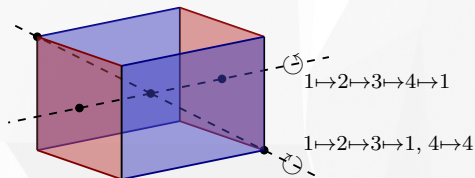
This quantum innovation chain materializes through collaborations with our friends at the University of Waterloo's Institute for Quantum Computing, the Quantum Valley Ideas Lab, Quantum Valley Investments, Communitech, and other nodes in the broader innovation space in the region.



What does the word “symmetry” really mean?

Symmetries are used throughout mathematics and physics.

A **symmetry** is an operation that preserves the “important” information while changing “unimportant” information. For example, symmetries in geometry preserve relationships between points (distances, angles, colinearity) while changing their actual locations. What counts as (un)important depends on the problem you are trying to solve.



The distance-preserving rotational symmetries of a cube form a **group** isomorphic to S_4 .

Symmetries in quantum physics can be noninvertible

The basic question of **physics** is to understand the relationship between small-scale microscopic descriptions and large-scale macroscopic descriptions of the same phenomena.

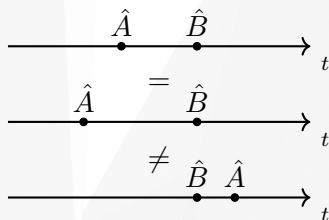
Often, the phenomena we care about are temporal processes: what will happen to a system? A **symmetry** is then an operator $\hat{A}$ which preserves the relationship “this state becomes that state.” In quantum mechanics, time evolution is described by the equation $\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$, where $|\psi\rangle$ is a vector and $\hat{H}$ is a matrix called the **Hamiltonian**. A **symmetry** is then an operator $\hat{A}$ s.t. $\hat{A}\hat{H} = \hat{H}\hat{A}$.

These are also called **constants of motion**. A symmetry is **unitary** if it also preserves the relationship “expectation to transition between states” i.e. the vector dot product $\langle - | - \rangle$. **But quantum theory includes superposition**. I will allow noninvertible superpositions of symmetry operators, getting an **algebra** rather than a **group**.

Symmetries operators are topological in time

In QM, a **symmetry** is an operator $\hat{A}$ s.t. $\hat{A}\hat{H} = \hat{H}\hat{A}$.

In other words, applying $\hat{A}$ at time t is the same as applying $\hat{A}$ at time $t + \epsilon$ for $\epsilon \approx 0$. But if you also apply operator $\hat{B}$ at time s , then applying $\hat{A}$ before or after time s can make a difference.



The **topology** of the configuration of points on the time-line matters, but the **geometry** of the configuration does not.

Symmetries operators are topological in spacetime

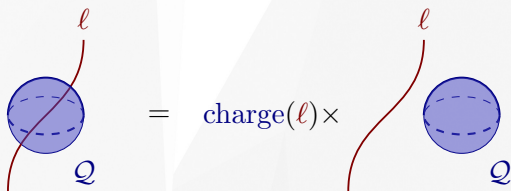
In quantum field theory, operations can occur at local moments in spacetime, or they can occur along extended surfaces. The evolution operator is a vector $\hat{P}_\mu$. Commuting with $\hat{P}_\mu$ moves the surface, so symmetries are again the **topological** operators:

Example: In pure electromagnetism, there is a **central symmetry** that changes the overall $U(1)$ phase of the gauge field. **Noether's Theorem:** This provides a local vector-valued $\star F_{\mu\nu}$, whose evolution in spacetime is $d_\lambda \star F_{\mu\nu} = [\hat{P}_\lambda, \star F_{\mu\nu}] = 0$. **Stokes' theorem:** The **surface operator** $\mathcal{Q}(\Sigma) = \int_{\Sigma^2} \star F$ is **topological**.



p -form symmetries and charges

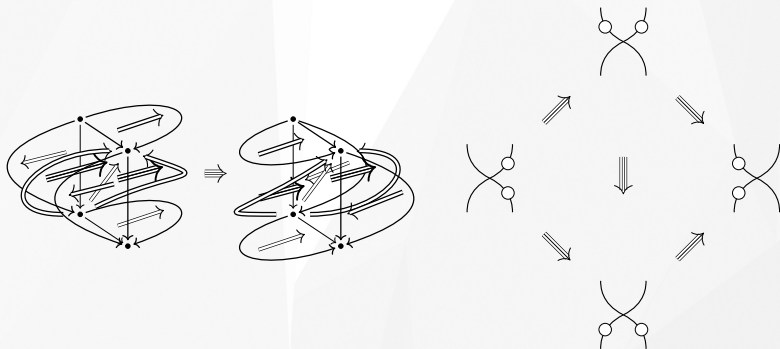
A topological surface operator like $\mathcal{Q}(\Sigma) = 2\pi i \theta \int_{\Sigma^2} \star F$ can move around points in 3+1D. But it gets stuck if you try to move it past a line. Example of a line operator: a nondynamical **test charge** moving through spacetime; the operator measures the work that the electromagnetic field would apply to that charge. The **charge** of a test particle is the eigenvalue of moving $\mathcal{Q}$ past the particle.



Because this $\mathcal{Q}$ charges 0+1D objects, it is called a **1-form symmetry**.

Mathematical organization: higher category theory

The mathematical language for organizing operations and their compositions is called **category theory**. When you have higher-dimensional operations between operations, you are doing **higher category theory**.



Global Categorical Symmetries: study global symmetries (not gauged, not infinitesimal) described by (higher) category theory.

Physics application of GCS: completeness of spectrum

Mathematicians allow stupid “symmetries” like “ $\mathbb{Z}/2\mathbb{Z}$ acting by $+1$.” In physics, we want to know the exact list of symmetries. How to know a symmetry is actually there? If there is a charged object, $\checkmark$. **Completeness of spectrum** posits that every actual symmetry has a corresponding charged object. **But this is false:** For example, there is no object charged under the “electromagnetic duality” symmetry. Proof: the electromagnetic duality symmetry can end topologically but noninvertibly; any object can just move around it.

Categorical completeness of spectrum: Don't use invertible symmetries modulo invertible equivalence. Use noninvertible symmetries modulo noninvertible symmetry interfaces.

Theorem (JF–Reutter): **Topological** quantum field theories (e.g. topological insulators, top'l quantum codes) satisfy categorical completeness of spectrum. **Nontopological case is still open.**

Physics application of GCS: Landau paradigm

If a microscopic description has some symmetry G , then the solutions might not be individually G -symmetry. Rather, they can be permuted by G . **Orbit-stabilizer Theorem:** For invertible symmetries, every G -orbit is isomorphic to G/H for some subgroup $H \subset G$, the **stabilizer** of the orbit. Physicists say that the G -symmetry **spontaneously breaks to H** if the solution is in a G -orbit isomorphic to G/H .

Many phases of matter are distinguished by their pattern of spontaneous symmetry breaking. **Landau paradigm** posits that patterns of SSB exactly classify phases. **But this is false:** The quantum hall phases are not classified by spontaneous breaking of invertible symmetries. **They are classified by spontaneous breaking of a noninvertible symmetry.**

Categorical Landau paradigm: Phases of matter are classifying by noninvertible SSB. **This is still open.**

Physics application of GCS: explaining neutrino masses?

Take a quantum field theory with a massless particle, e.g. neutrino in the standard model. Perturb the Lagrangian a little bit to add a UV mass term. Typical behaviour: the mass term will “run,” meaning that the effective mass measured at macroscopic scales will be large. What is going on when masses measured at large scales are small but not zero, e.g. neutrino masses?

Soft symmetry breaking: If the massless theory has a symmetry which the perturbation breaks just a little bit, then the symmetry can prevent the mass from running. But the standard model does not have a symmetry whose soft breaking explains neutrino masses.

Physics theorem (Córdova–Hong–Koren–Ohmori): The standard model with the $\mathbb{Z}/3\mathbb{Z}$ flavour symmetry gauged does have a noninvertible symmetry, whose soft breaking explains small neutrino masses in that model.

Higher categories as noninvertible topological spaces

Group Theory $\subset$ Algebraic Topology: any group can be encoded as the paths in a topological space. For example, $\mathbb{R}P^2$ encodes the group with presentation $\langle g \mid g^2 = 1 \rangle$, aka $\mathbb{Z}/2\mathbb{Z}$:

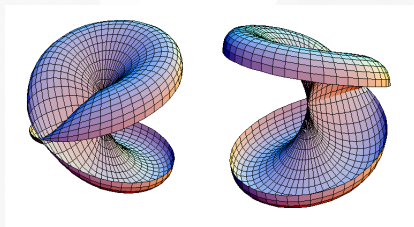


Image from Wikipedia

Noninvertible symmetries — higher categories — require **noninvertible algebra topology**. We are just at the beginning of developing it.



THANK YOU

25
YEARS

PI

PERIMETER
INSTITUTE